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Optimization of dimension of weldment locus by method of geometric programming

ABSTRACT

This paper, on the example of a typical loaded welded assembly, made optimization of its dimensions in terms of the cost of welding. In such an elaboration the mathematical optimization model with limitation functions has also been presented and it should be taken into account in the process of designing by the technologist and designer.

To solve the presented problem the method of geometric programming was proposed that has in detail been elaborated in the paper in the form of an algorithm suitable for the application. In this way the optimization or primary task was reduced to a dual task through a proper function, which is much easier to solve.

The method has been illustrated on a practical computational example with a different number of limitation functions. It is shown that in case of a lower degree of complexity the solution can be reached by maximizing the corresponding dual function by means of mathematical analysis. In case of a higher degree of complexity, it is necessary to use some of the methods of non-linear programming. In this case the solution of the problem is simplified due to the minimization of a linear equation.

Keywords: *Alphabetic loaded welded structures , the mathematical model of optimization, the cost function , feature limitations , geometric programming, positive polynomials, dual function.*

1. MATHEMATICAL MODEL OF OPTIMIZATION

Mathematical basis of techno-economic optimization of the objects is the mathematic model of optimization. Model of optimization according to figure 1 consists of the components:

- State functions F_{si} (i = 1,2,3 ...),
- Limit function (function boundary conditions) F_{gi} (i = 1,2,3 ...),
- Criteria optimization and
- The objective function F_{ci} (i = 1,2,3 ...).

The first two components, ie. state functions or state equation and the limit function object of the mathematical model of the object is defined. Real objects imply a wide range of phenomena, processes and systems as very frequent objects of modelling in mechanical engineering and machining as technological processes [1,2].

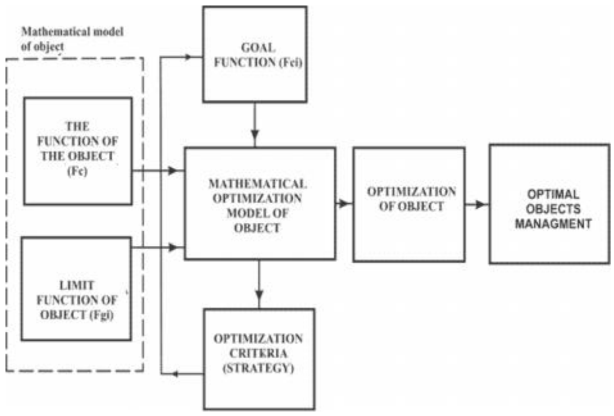

Figure 1 - The structure of the mathematical model of optimization object

It should be noted that the mathematical model, as opposed to the physical retains the physical nature of the originals (real property), showing the mathematical abstraction. This abstract form expresses the essential physical, geometrical, technological, economic or any other features of the real object, [3,4].

The mathematical model of a machining process can be generally shown schematically by Figure 2.

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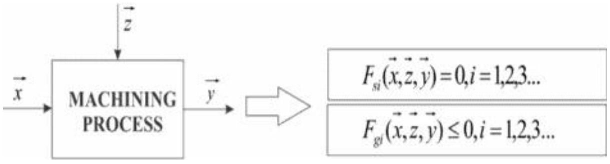

Figure 2 - The mathematical model of the machining process

In this, it is necessary to analyze the inputs and outputs of "natural" process and all sets of input-output variables x_i, y_i, z_i i = 1,2,3 ... for each decomposed "elementary" process. This is a "natural" process that constitutes the core of the machining process and should describe the mathematical model of the process [5-7].

After analyzing the existing restrictions one accesses the mathematical description of the object in the mathematical language as a specific set of functions and equations. Thus the main features of the mathematical model are a function of the state of F_{si} and function limitations F_{gi}.

Considering the scheme in Figure 2, we can set up a mathematical model of any machining process in production engineering (with and without removing the chip, and beyond), through the functions of the process:

$$F_{si} = (\vec{x},\vec{y},\vec{z}) = 0\,,\ i = 1,2,3..., \tag{1}$$

and the limit function:

$$F_{gi} = (\vec{x},\vec{y},\vec{z}) \le 0 \quad i = 1,2,3..., \tag{2}$$

The mathematical models (1) and (2) are essentially physical, geometrical, technological and economic dependent qualimetric within the machining process and to the admissible domain D resizing.

The system (1, 2) vectors $\vec{x},\vec{y},$ and $\vec{z}$ denote a set of variables input- output variables of the process . Vector characteristics of the process states or controlled size

$$\vec{y} = (y_1, y_2, y_3, ... y_n),$$

describe the state and behavior of the machining process and the system was created as a consequence of inputs $\vec{x}$, and $\vec{z}$. Input parameters, which are numerous, are divided into controlled $(\vec{x})$ and uncontrolled $(\vec{z})$ size of the process. The vector $\vec{x}$, includes all inputs to the process whose value can be measured numerically. The vector $\vec{x} = (x_1, x_2, x_3, ... x_p)$, can be broken down into a group optimal or control the size and groups that are constant in the course of the process. The first group can be changed in the process in order to achieve the desired state of the process $(\vec{y})$ and optimum objective function (F_c).

Vector uncontrolled size $\vec{z} = (z_1, z_2, z_3, ...)$ contains one input parameters whose values can not be measured, as well as those that can be measured, but whose impact on the negligibly small. Vector causes adverse conditions and adverse changes in the flow characteristics of the process $(\vec{y})$ or the objective function (F_c).

For the case of deterministic processes impact of uncontrolled factors $\vec{z}$ is not large, there is a correlation between the characteristics of states $\vec{y}$ and inputs $\vec{x}$. For this model, the size $\vec{z}$ will not be contained in the relations (1) and (2) , so that it is obtained:

$$F_{si} = (\vec{x},\vec{y}) = 0 \ \ i = 1,2,3..., \tag{3}$$

$$F_{gi} = (\vec{x},\vec{y}) \le 0 \ \ i = 1,2,3..., \tag{4}$$

or in the form of an explicit,

$$\vec{y} = F(\vec{x}), \tag{5}$$

Components of the state functions and function limitations, as it is said, is defined as the mathematical model while the optimization criteria as the third component, together with the first two, sets the framework mathematical model of optimization. On the basis of these three components, there is a specific form of the objective function (optimization functions):

$$F_c = F_c(\vec{x},\vec{y},\vec{z}), \tag{6}$$

Function (6) is a mathematical description of the optimal control process, the identified optimization criteria.

In theory, techno-economic optimization can be extracted more optimization criteria or objective functions (F_c) according to which the optimized processes, [8,9,2,10]:Cost (T_i), Build time (T_{ui}), Economy (E_i), Productivity (P_i), Profitability (R_i), Quality (K_i).

Bringing to criteria optimization

$$F_{ci} = (T_i, t_{ui}, E_i, P_i, R_i, K_i ...), \tag{7}$$

which shows key production effects, and also functions F_{gi} constraints, which limit the allowed domain D changes of input, into a functional relationship with a set of input and the other mentioned factors, according to (6) and (2), is obtained by the mathematical model of optimization deterministic process:

$$F_c = F_c(x_1, x_2, x_3, ... x_p), \tag{8}$$

$$x \in D \quad D\begin{cases} x_i = c_i \\ a_r \le x_i \le b_r \\ F_{gj}(x_1, x_2, x_3, ... x_D) \le 0 \end{cases}$$

Techno-economic optimization is reduced to a mathematical point of view, the definition of extremal (optimal) objective function (8) and of correspondingof managed size and characteristics of the state of the process that provides the optimum, as shown in Figure 3, [1,3,11].

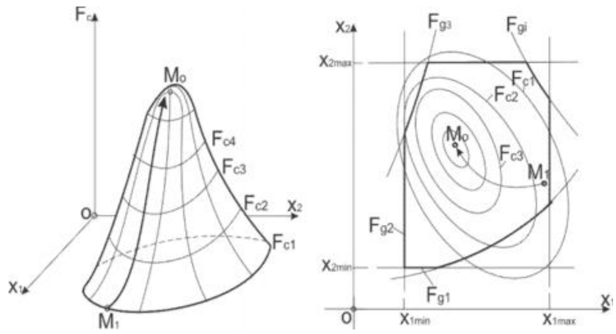

Figure 3 - Diagram of surface features to optimize the optimal range

The optimal level or solution $\vec{x}_0 = (x_{10}, x_{20}, x_{30}, ... x_{k0})$ of the objective function (8) is called a local extremum, and point M_0 (defined vector $\vec{x}_0$) is called a point of local extremum. Function F_c may have several local extremes.

To form the mathematical model of optimization according to (8), in addition to mathematical expressions objective function F_c, it is necessary to set up a mathematical expression of all necessary functions restrictions F_{gi}, $i = 1,2,3,...$ This way, defines and limits of permissible or the optimal area, region or domain D. All these constraints can be expressed in the form of equations and inequalities containing, among others, and given the size of sets of input values x_i, [8,12]

According exposed, it can be concluded that the mathematical model of optimization, no matter what the subject is the word (process, system, structure, management, etc.) must always contain, as in the case of machining process, four basic components: Function of the facility, Limit function, The criteria optimization, Function of optimization or objective function.

Based on these components, forming the final shape of the model optimization of a given object that expresses the function optimization $F_c = F_c(x_i)$ and the permitted domain D transformations of variables x_i.

2. THE METHOD OF GEOMETRIC PROGRAMMING

This method can solve those optimization tasks whose optimization functions are in the form of positive polynomials:

$$F_c(x) = \sum_{j=1}^{k} B_j \sum_{i=1}^{k} x_i^{b_{ij}}, \tag{9}$$

where: B_j -positive coefficients (constants), bij-exponents, random number, which may be positive, negative or zero value, x_i-independent variables (variables) that can only have positive values.

The algorithm of method, which will be summarized in the following, allows to determine the optimal solution $\vec{x}_0 = (x_{10}, x_{20}, x_{30}, ... x_{k0})$ with the $F_{c\ \min}$, [13-16].

In many cases, the optimization of machining processes and technology in terms of cost, where the optimization function is expressed as a positive polynomial (9), it is possible to effective application of the method of geometric programming.

2.1. The basic inequality methods

In developing the algorithm method of geometric programming, starting from the mathematical inequality between geometric and arithmetic means of non-negative numbers. This inequality is the foundation of the method and two sizes as follows, [16,12,17,3,4].

$$\frac{1}{2}(Z_1 + Z_2) \geq \sqrt{Z_1 Z_2}, \tag{10}$$

This relation expresses the view that geometry can not be greater than arithmetic means.

Inequality (10), for k variables is:

$$\sum_{j=1}^{r} q_j Z_j > \prod_{j=1}^{r} Z_j^{q_j} \tag{11}$$

and it is valid that the size Z_j positive and positive size q_i satisfy the condition of normality,

$$\sum_{j=1}^{r} q_j = 1 \tag{12}$$

From equation (11), we can write the basic equations of the method of geometric programming:

$$\sum_{j=1}^{r} z_j \geq \sum_{j=1}^{r} \left(\frac{z_j}{q_j} \right)^{q_j}, \tag{13}$$

The previous equation is obtained when the inequality in replacing,

$$z_j = q_j Z_j, \tag{14}$$

where in a $z_j > 0$.

Inequality (14) has a fundamental meaning for the method of geometric programming because the application of this inequality to the function optimization (9) may change:

$$z_j(x) = B_j \prod_{i=1}^{k} x_i^{b_{ij}}, \tag{15}$$

in equation (13), write the basic of mathematical method

$$\sum_{j=1}^{r} B_j \prod_{i=1}^{k} x_i^{b_{ij}} \geq \prod_{j=1}^{r} \left(\frac{B_j}{q_j}\right)^{q_j} \prod_{i=1}^{k} x_i^{L_i}, \quad (16)$$

where in,

$$L_i = \sum_{j=1}^{r} q_j b_{ij}, \quad (17)$$

It was pointed out that inequality (16) is valid for any positive values of size q_i, which must satisfy the condition of normality (12). Proceeding from this, we can simplify the inequality (16) the choice of values for q_i to obtain L_i = 0 in equation (17), ie.

$$L_i = \sum_{j=1}^{r} q_j b_{ij} = 0 \quad i = \overline{1,k}, \quad (18)$$

Equation (*X*) simplified the fundamental expression (*X*), which now reads:

$$\sum_{j=1}^{r} B_j \prod_{i=1}^{k} x_i^{b_{ij}} \geq \prod_{j=1}^{r} \left(\frac{B_j}{q_j}\right)^{q_j}, \quad (19)$$

This equation applies to the condition of normality:

$$\sum_{j=1}^{r} q_j = 1, \quad (20)$$

orthogonality, in accordance with equation (17), ie.

$L_i = 0$

$$\sum_{j=1}^{r} q_j b_{ij} = 0 \quad i = \overline{1,k}, \quad (21)$$

and the condition of positivity:

$$q_j > 0 \quad i = \overline{1,r}, \quad (22)$$

Right side of the inequality (19) is a function of the size of q_j ($i = \overline{1,r}$), as can be seen, ie.

$$Q(q) = \prod_{j=1}^{r} \left(\frac{B_j}{q_j}\right)^{q_j}, \quad (23)$$

and is called a dual function of a convex function (9), because the positive polynomial (9) is a convex function.

Left F_c inequality (19), however, depends only on the independent variables x_j ($i = \overline{1,r}$).

The following conclusion is: at the basis of the fundamental inequality , a polynomial of positive F_c *(X)* cannot be, whatever kind of values are the variables x_j ($i = \overline{1,r}$),smaller than the dual function *Q (q), (X)*, and the primary model, i.e. minimization function F_c, down to the dual model, ie. the maximization of the dual function *Q(q).* So there is a primary reformulation (base, starting) in finding the optimization dual task, since it can be shown [15,7,16] that is a maximum value of the dual function *Q(q)* equal to the minimum value of the basic functions in the form of a positive F_c polynomials, ie.

$$\min F_c(x) = \max \sum_{j=1}^{r} \left(\frac{B_j}{q_j}\right)^{q_j}, \quad (24)$$

In these conditions must be met (20), (21) and (22) of the dual variable Q_j ($i = \overline{1,k}$). So optimization (primary) task,

$$F_{c\,min} = \min \sum_{j=1}^{r} B_j \prod_{i=1}^{k} x_i^{b_{ij}} = F_{c0}, \quad (24a)$$

$$x_i > 0$$

$$Q(q)_{\max} = \max \prod_{j=1}^{r} \left(\frac{B_j}{q_j}\right)^{q_j} = Q_0, \quad (25)$$

$$\sum_{j=1}^{r} q_j = 1 \quad \sum_{j=1}^{r} q_j b_{ij} = 0 \quad i = \overline{1,k} \quad (25a)$$

$$q_j > 0 \quad i = \overline{1,r}$$

which is much easier to solve in respect of the primary task (25), i.e. determination of $F_{c\ min}$, F_c optimization function.

2.2. Algorithm of method

At present, there are two possible cases as follows:

- The case without restrictions
- The case of the constraints

In case that there are no limitation equations (24) or system (25) is defined by the minimum value (optimum) function optimization F_{c0} (9) over the maximum Q_0, ie. determining the optimal level F_{c0} positive polynomial F_c testifies to the determination of the maximum value Q_0 dual function *Q(q)*. It would be the first step of the method.

In the second step is calculated the optimal dual vector $\vec{q}_0 = (q_{10}, q_{20}, q_{30}, ...q_{k0})$ system of equations:

$$\sum_{j=1}^{r} q_j = 1 \quad \sum_{j=1}^{r} q_j b_{ij} = 0 \quad i = \overline{1,k}, \quad (26)$$

which represents the conditions of normality and orthogonality.

In the third step is determined the maximum value Q_0 dual function $Q(q)$, which is based on a known set $\vec{q}_0$ (determined in the second step), as calculated from the equation:

$$Q_0 = max \prod_{j=1}^{r} \left(\frac{B_j}{q_{j0}} \right)^{q_{j0}}, \quad (27)$$

The fourth step involves the determination of optimal primary vector (solution) $\vec{x}_0 = (x_{10}, x_{20}, x_{30}, ... x_{k0})$.

The functions of the F_c, (9) found on the basis of the values F_{c0}=Q_0 in the third step. Between $\vec{x}_0$ that minimizes the function F_c = F_0, the optimal dual vector $\vec{q}_0$ which satisfies the conditions of normality and orthogonality (26), the second step becomes as follows:

$$Q_0 q_0 = B_j \prod_{i=1}^{k} x_{i0} b_{ij} \quad i = \overline{1, r}, \quad (28)$$

From equation (28), we obtain the required optimal level $\vec{x}_0$, where the size of Q_0 = F_{c0} and q_{j0} ($i = \overline{1, r}$) are known, as well as some of the second or third step.

In case you are given the limitations in the form of the function:

$$F_{g1} \leq 1,\ F_{g2} \leq 1,\ F_{g3} \leq 1, ..., \quad (29)$$
$$F_{gp} \leq 1$$

then the primary task reads

$$F_{cmin} = min \sum_{j=1}^{r} F_j \prod_{i=1}^{k} x_i^{b_{ij}}, \quad (30)$$

$$F_{g1} \leq 1,\ F_{g2} \leq 1,\ F_{g3} \leq 1, ...,$$
$$F_{gp} \leq 1\ x_1 > 0,\ x_2 > 0,\ x_3 > 0, ..., \quad (31)$$
$$x_k > 0$$

wherein the function restriction F_{gt} ($t = \overline{1, p}$) has the shape of a positive polynomials

$$F_{gt} = min \sum_{j \in J(t)} B_j \prod_{i=1}^{k} x_i^{b_{ij}}, \quad (32)$$
$$t = 1,2,3,..., p$$

Where in:

J (0) (1,...,m_0) - indexes of individual members function F_c

J (1) (m_0 +1 ...,m_1) - indexes of individual members function f_{g1}

J (2) (m_1 +1, ...,m_2) - indexes of individual members function F_{g2}...

J (p) (m_{p-1} +1, ..., m_p = r) - indexes of individual members function F_{gp},

Here, the functions F_c and F_{gp} are in the shape of a positive polynomial.

With corresponding dual task, in which the primary task is reduced, we can show and express the system:

$$Q(q)_{max} = max \left[\prod_{j=1}^{r} \left(\frac{B_j}{q_j} \right)^{q_j} \right] \prod_{t=1}^{p} a_t^{a_t}, \quad (33)$$

$$a_t = \sum_{j \in J(t)} q_j \quad t = 1,2,3,..., p, \quad (34)$$

$$\sum_{j=1}^{r} q_j = 1\ \sum_{j=1}^{r} q_j b_{ij} = 0 \quad i = \overline{1, k}, \quad (35)$$

$$q_j > 0 \quad i = \overline{1, r}, \quad (36)$$

As we see from (32) see in equation (33) are entered all coefficients B_j containing F_c function and system function limitations F_{gt} ($t = \overline{1, p}$). Here, conditions (35) and (36) are conditions of normality, orthogonality and positivity, respectively.

The further course of the optimization algorithm given object is identical to the method of geometric programming algorithm without constraints. However, the equations (28) to determine the optimum number of the primary vector in this case is modified to read as follows:

$$B_j \prod_{i=1}^{k} x_i^{b_{ij}} = \begin{cases} Q_0 q_{j0}\ za\ j \in J(0) \\ \dfrac{q_{j0}}{a_{t0}}\ za\ j \in J(t)\ \ t = \overline{1, p} \end{cases}, \quad (37)$$

wherein:

$$a_{t0} = \sum_{j \in J(t)} q_{j0}, \quad (38)$$

3. CALCULATION EXAMPLES

The general task of optimizing the mathematical model of the illustrated two examples of the structural unit is related to the optimization of dimensions of the welded assembly loaded in terms of the welding costs.

3.1. A simple example

In the first example, Figure 4 circuit consists of two elements: beams (girders) 1 with weld and hokder (reliance, where the beam is fixed to a rigid bracket welded weld I and II.

a) Definition of fixed and variable size

According to the exposed the next procedure must be defined first as well as unchangeable variable resolution image. Conditions of the problem are given constant (unchangeable) size: kinds of materials of manufacturers, the free length of the beam (units) L and the maximum force F on loaded beams.

Other dimensions of the assembly are independent variables. These dimensions are:

$$h = x_1 \; l = x_2 \; t = x_3 \; b = x_4, \quad (39)$$

The value of these dimensions should be so determined and optimized to achieve an optimal vector $\vec{x}_0 = (x_{10} = h_0, x_{20} = l_0, x_{30} = t_0, x_0 = b_0)$ of minimum cost of welding. Fcmin = Fco = min T.

b) Defining the mathematical form of function optimization

The cost function as a function of optimization can be written as, [18-22].

$$T = T_p + T_1 + T_2, \quad (40)$$

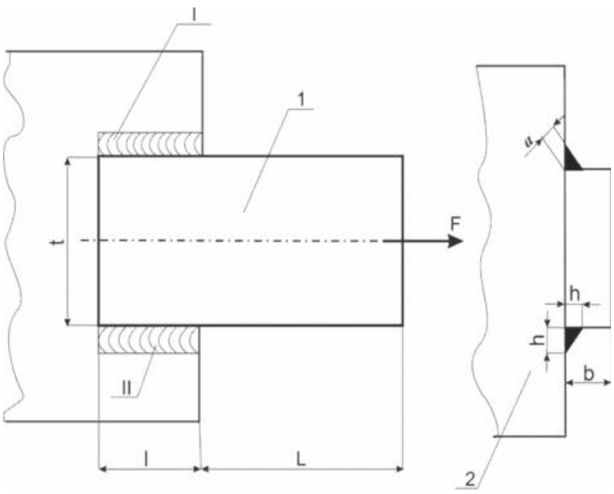

Figure 4 - Loaded weldments

These functions consist of three main components (partial charges): T_p - the costs of preparation (preparatory operations),T_1-welding costs,T_2-the cost (price) of material.

Costs of preparation T_p refer to all the necessary technological equipment to perform welding operations: welding tools, auxiliary equipment for setting beams on the truss in position, its tightness and more. These costs will be considered constant (do not depend on the variables (x_1, x_2, x_3, and x_4).

Cost of welding operations T_1 can be determined if you know the elements of these costs:

T_{11} - the cost of using welding expressed in monetary amount per unit of time, which includes the cost of amortization and loan repayment appliances, the cost of auxiliary equipment (depreciation) used in welding, the cost of human work (personal income from contributions and other),

Q_z - The capacity ie. volume of weld (weld) per unit time and,

V_z - volume of weldment, weld I and II, that the example given is calculated as:

$$V = V_{z1} + V_{z2} = \frac{1}{2}h^2 l + \frac{1}{2}h^2 l = h^2 l, \quad (41)$$

as follows according to figure 4.

On the basis of these elements can be written T1 costs:

$$T_1 = \frac{T_{11}}{Q_z} V_z = \frac{T_{11}}{Q_z} h^2 l, \quad (42)$$

The cost of materials will be:

$$T_2 = T_3 V_z + T_4 V_G, \quad (43)$$

Where in: T_3 - material price of weld, T_4 - material price beam, V_g - volume of the beam is calculated as:

$$V_G = t \cdot b \cdot (b + l), \quad (44)$$

replacing (41) and (44) to (43) will be:

$$T_2 = T_3 \cdot h^2 l + T_4 \cdot t \cdot b \cdot (L + l), \quad (45)$$

Costs by replacing (42) and (45) in (40) we obtain the desired shape optimization function (objective function):

$$F_c = T = T_p + \frac{T_{11}}{Q_z} \cdot h^2 l + T_3 \cdot h^2 l + $$
$$+ T_4 \cdot t \cdot b \cdot (L + l), \quad (46)$$

respectively:

$$T = T_p + \left(\frac{T_{11}}{Q_z} + T_3 \right) \cdot h^2 l + T_4 \cdot t \cdot b \cdot (L + l), \quad (47)$$

or by (39):

$$T = T_p + \left(\frac{T_{11}}{Q_z} + T_3 \right) \cdot x_1^2 \cdot x_2 + T_4 \cdot x_2 \cdot x_3 \cdot x_4 + $$
$$+ T_4 \cdot L \cdot x_3 \cdot x_4, \quad (48)$$

the present values of the coefficients T_{11}, Q_z, T_3, T_4 and L-known of the given task (objective optimization).

c) Defining and setting up a system function limitations

1. Restrictions on the power of the shear in the weld [21-23].

The actual shear stress in the weld will be, in view of the computational of weld $a = \frac{h\sqrt{2}}{2}$, Figure 4.

$${}^2 = {}^2(x_i) = \frac{F}{2 \cdot A_z} = \frac{F}{2 \cdot a \cdot l} = \frac{F}{h\sqrt{2} \cdot l} = $$
$$= \frac{F}{\sqrt{2} \cdot x_1 \cdot x_2} \le {}^2{}_d, \quad (49)$$

For allowable tension shear ${}^2{}_d$, will apply to:

$$^{2}{}_{d} \geq \frac{F}{\sqrt{2}\cdot x_1 \cdot x_2} \quad ^{2}{}_{d} \geq {}^{2}(x_i), \qquad (50)$$

Dividing equation (50) to be:

$$1 \geq \frac{F}{^{2}{}_{d}\cdot\sqrt{2}\cdot x_1 \cdot x_2}, \qquad (51)$$

or as a function of the limits being:

$$F_{g1} = \frac{F}{^{2}{}_{d}\cdot\sqrt{2}} x_1^{-1}\cdot x_2^{-1} \leq 1, \qquad (52)$$

Shape of function F_{g1} and other function of optimization in this form, as will be seen that it is suitable for optimization.

2. Restrictions on the normal stress stretch material of manufacturers, [21,22,24].

The actual tension will be less than the allowable:

$$\pm(x_i) = \frac{F}{A} = \frac{F}{t\cdot b} \leq \pm_d, \qquad (53)$$

respectively:

$$\frac{F}{t\cdot b\cdot \pm_d} \leq 1, \qquad (54)$$

given the limitations of the function being:

$$F_{g2} = \frac{F}{t\cdot b\cdot \pm_d} = \frac{F}{x_3\cdot x_4\cdot \pm_d} = \\ = \frac{F}{\pm_d} x_3^{-1}\cdot x_4^{-1} \leq 1, \qquad (55)$$

3. Restrictions related to the practical possibility of getting welds

This limit is expressed as, $b \geq h$,as the beam width must be greater than the weld parameter h. It follows that:

$$x_4 \geq x_1 \; 1 \geq \frac{x_1}{x_4}, \qquad (56)$$

Given the limitations of the function being:

$$F_{g3} = \frac{x_1}{x_4} \leq 1 \;\; F_{g3} = x_1\cdot x_4^{-1} \leq 1, \qquad (57)$$

4. Restrictions on the non-negativity variables x_i.

This limitation is expressed by the function:

$$F_{g4} = x_i \geq 0 \qquad (58)$$

d) A mathematical model of optimization

According to exposed relations (48), (52), (55), (57), (58) for the observed structural structure, the mathematical model of optimization will be:

$$F_c = T = \\ \min\left[\left(\frac{T_{11}}{Q_z} + T_3\right)\cdot x_1^2\cdot x_2 + T_4\cdot x_2\cdot x_3\cdot x_4 + T_4\cdot L\cdot x_3\cdot x_4\right], \qquad (59)$$

$$D\begin{cases} F_{g1} = \frac{F}{^{2}{}_{d}\cdot\sqrt{2}} x_1^{-1}\cdot x_2^{-1} \leq 1 \\ F_{g2} = \frac{F}{\pm_d} x_3^{-1}\cdot x_4^{-1} \leq 1 \\ F_{g3} = x_1\cdot x_4^{-1} \leq 1 \\ F_{g4} = x_i \geq 0 \end{cases}, \qquad (60)$$

$i = 1,2,3,4$

The function (59), the cost of preparation T_p as a constant for the observed relation is not taken into account since they do not affect the mathematical analysis that follows. Once the minimum function of the F_c, the same value must only add the cost of the preparation, with respect to the relation (40).

By introducing the (constant):

$$T_{13} = \frac{T_{11}}{Q_z} + T_3$$

$$F_a = \frac{F}{\sqrt{2}\cdot{}^{2}{}_{d}} \;\; F_b = \frac{F}{\pm_d}, \qquad (61)$$

Relation (59) and (60) are simplified:

$$F_c = T = \min\left[T_{13}\cdot x_1^2\cdot x_2 + T_4\cdot x_2\cdot x_3\cdot x_4 + T_{4L}\cdot x_3\cdot x_4\right]$$

$$F_{g1} = F_a\cdot x_1^{-1}\cdot x_2^{-1} \leq 1,\; F_{g3} = x_1\cdot x_4^{-1} \leq 1 \qquad (62)$$

where in:

$x_1 \geq 0$; $x_2 \geq 0$; $x_3 \geq 0$; $x_4 \geq 0$

According to the algorithm in chapter 2.2. for the case that there are limits, the corresponding dual function, considering to (62), will be:

$$Q(q) = \left(\frac{T_{13}}{q_1}\right)^{q_1}\cdot\left(\frac{T_4}{q_2}\right)^{q_2}\cdot\left(\frac{T_{4L}}{q_3}\right)^{q_3}\cdot\left(\frac{F_a}{q_4}\right)^{q_4} \\ \cdot\left(\frac{F_b}{q_5}\right)^{q_5}\cdot\left(\frac{1}{q_6}\right)^{q_6}\cdot q_4^{q_4}\cdot q_5^{q_5}\cdot q_6^{q_6}, \qquad (63)$$

after the task has a total of six members: r = 6, and three in the F_c and three in the F_g, since there are three function limitation (t = 1) each having a single member.

From the condition of normality (35) and (36) and orthogonal forms a system of five equations with six unknown:

$$\begin{aligned} &(I) \quad q_1 + q_2 + q_3 = 1 \\ &(II) \quad 2q_1 - q_4 + q_6 = 0 \\ &(III) \quad q_1 + q_2 - q_4 = 0 \\ &(IV) \quad q_2 + q_3 - q_5 = 0 \\ &(V) \quad q_2 + q_3 - q_5 - q_6 = 0, \end{aligned} \qquad (64)$$

Obviously, I equation is a condition of normality (Fc function has three members, and therefore appears (q_1, q_2, q_3), while the other equation (II-V) are orthogonality, in order to variable x_1, x_2, x_3 and x_4.

In this equation (II) is determined by x_1, the equation (III) by x_2, the equations (IV) to x_3, and the equation (V) according to x_4, taking into account their exponents. Total number of q_i (i = 1-6) is equal to the number of members of the F_c function and the function limitation (3 +1 +1 +1 = 6).

Equation V by subtracting equation IV, it follows that:

$$q_6 = 0, \tag{65}$$

By using the Gaussian algorithm, a simple way to show that all of the unknowns can be expressed in terms of q_1.

From the second equation it follows that:

$$q_4 = 2q_1, \tag{66}$$

as follows from the III:

$$q_2 = q_1, \tag{67}$$

At the end of the equation I and IV it follows that:

$$\begin{aligned} q_3 &= 1-2q_1 \\ q_5 &= 1-q_1, \end{aligned} \tag{68}$$

Rearranging equation (63), it will be simplified:

$$Q(q) = \left(\frac{T_{13}}{q_1}\right)^{q_1} \cdot \left(\frac{T_4}{q_2}\right)^{q_2} \cdot \left(\frac{T_{4L}}{q_3}\right)^{q_3} \cdot F_a^{q_4} \cdot F_b^{q_5} \cdot 1^{q_6}, \tag{69}$$

substituting q_2, q_3, q_4, q_5, q_6, and by (65), (66), (67) and (68), equation (69) becomes:

$$Q(q) = \left(\frac{T_{13}}{q_1}\right)^{q_1} \cdot \left(\frac{T_4}{q_1}\right)^{q_1} \cdot \left(\frac{T_{4L}}{1-2q_1}\right)^{1-2q_1} \cdot F_a^{2q_1} \cdot F_b^{1-q_1}, \tag{70}$$

Obviously, the dual function $Q(q)$ is expressed in more than q_1, which was the goal.

Logarithmic functions (70) will be:

$$\begin{aligned} \ln Q(q) &= q_1 \ln\left(\frac{T_{13}}{q_1}\right) + q_1 \ln\left(\frac{T_4}{q_1}\right) + (1-2q_1)\cdot \\ &\cdot \ln\left(\frac{T_{4L}}{1-2q_1}\right) + 2q_1 \ln F_a + (1-q_1)\ln F_b, \end{aligned} \tag{71}$$

Let $\overrightarrow{q_0} = q_{j0}$, $j = \overline{1,6}$, the stationary point of the vector in which the $Q(q)_{max}$=$Q_{0\ max}$, then the same count is achieved and maximum functions $\ln Q(q)$, according to the (71).

So to calculate the derivative of the function $\ln Q(q)$ the variable q_1 and equates it to zero:

$$\frac{d}{dq_1}\left[\ln Q(q)\right] = 0, \tag{72}$$

Given that this is a complex function, for simplification to (71), we can introduce shifts:

$$\begin{aligned} Q_1 &= q_1(\ln T_{13} - \ln q_1) = q_1 \ln T_{13} - q_1 \ln q_1 \\ Q_2 &= q_1 \cdot \ln\left(\frac{T_4}{q_1}\right) \\ Q_3 &= (1-2q_1)\cdot \ln\left(\frac{T_{4L}}{1-2q_1}\right) = \ln\left(\frac{T_{4L}}{1-2q_1}\right) - \\ &- 2q_1 \ln\left(\frac{T_{4L}}{1-2q_1}\right) \\ Q_4 &= 2q_1 \ln F_a \\ Q_5 &= (1-q_1)\cdot \ln F_b = \ln F_b - q_1 \ln F_b \end{aligned} \tag{73}$$

With this shift, the function (71) becomes:

$$\ln Q(q) = Q_1 + Q_2 + Q_3 + Q_4 + Q_5, \tag{74}$$

Derivative of (74) will be:

$$\frac{d\left[\ln Q(q)\right]}{dq_1} = Q_1' + Q_2' + Q_3' + Q_4' + Q_5', \tag{75}$$

Partial derivative of functions (75) for q_1 will be to (73):

$$Q_1' = \ln T_{13} - (\ln q_1 + 1),\ Q_2' = \ln\frac{T_4}{q_1} - 1 \tag{76}$$

$$Q_3' = \frac{2}{1-2q_1} - 2\cdot \ln T_{4L} + 2\cdot \ln(1-2q_1) - \frac{4\cdot q_1}{1-2q_1}$$

$$Q_4' = 2\cdot \ln F_a,\quad Q_5' = -\ln F_b$$

Substituting extracts partial functions (76) in (75) will be after the arranging, according to (72):

$$\begin{aligned} &\frac{2}{1-2q_1} - \frac{4q_1}{1-2q_1} + \ln\left(\frac{T_4}{q_1}\right) + 2\cdot \ln(1-2q_1) + \\ &+ \ln\left(\frac{T_{13}}{q_1}\right) - 2 - 2\ln T_{4L} + \ln\frac{F_a^2}{F_b} = 0, \end{aligned} \tag{77}$$

Equation (77), after some mathematical operations can be summarized as:

$$\ln\left(\frac{1-2q_1}{q_1}\right)^2 + \ln\left(\frac{T_{13}\cdot T_{4L}\cdot F_a^2}{T_4^{\,2}\cdot F_b}\right) = 0, \tag{78}$$

It follows that:

$$\left(\frac{1-2q_1}{q_1}\right)^2 = \frac{T_{4L}^2 \cdot F_b}{T_{13}\cdot F_a^2 \cdot T_4}, \tag{79}$$

and finally, solving to $q_1 \equiv q_0$:

$$q_{10} = \frac{1}{2 + \dfrac{T_{4L}}{F_a}\cdot\sqrt{\dfrac{F_b}{T_{13}\cdot T_4}}}, \tag{80}$$

Taking into account (65), (66), (67), (68) and (80) it follows that:

$$q_{20} = q_{10}$$

$$q_{30} = 1 - 2 \cdot q_{10} = 1 - \frac{2}{2 + \frac{T_{4L}}{F_a} \cdot \sqrt{\frac{F_b}{T_{13} \cdot T_4}}}$$

$$q_{40} = 2 \cdot q_{10} = \frac{2}{2 + \frac{T_{4L}}{F_a} \cdot \sqrt{\frac{F_b}{T_{13} \cdot T_4}}},$$

$$q_{50} = 1 - q_{10} = 1 - \frac{1}{2 + \frac{T_{4L}}{F_a} \cdot \sqrt{\frac{F_b}{T_{13} \cdot T_4}}} \qquad (81)$$

$$q_{60} = 0$$

Accordingly, an optimal dual vector has a component:

$$\vec{q}_0 = (q_{10}; q_{20}; q_{30}; q_{40}; q_{50}; q_{60}), \qquad (82)$$

By setting the calculated optimum dual component vectors $\vec{q}_0$ (82) corresponding to a maximum of the dual function of (25)

$$Q(q)_{max} = max\, Q(q) = Q_0 = \\ = Q(q_{10}; q_{20}; q_{30}; q_{40}; q_{50}; q_{60}), \qquad (83)$$

receives the value of the minimum function optimization, ie.:

$$F_{c0} = min\, F_c = max\, Q(q) = Q(q_0) = Q_0, \qquad (84)$$

Based on F_{c0}, calculated from equation (62) to (37) components of the optimal vector of the system:

$$(I) \quad T_{13} \cdot x_{10}^2 \cdot x_{20} = Q_0 \cdot q_{10}$$

$$(II) \quad T_4 \cdot x_{20} \cdot x_{30} \cdot x_{40} = Q_0 \cdot q_{20}$$

$$(III) \quad T_{4L} \cdot x_{30} \cdot x_{40} = Q_0 \cdot q_{30}$$

$$(IV) \quad F_a \cdot x_{10}^{-1} \cdot x_{20}^{-1} = \frac{q_{40}}{a_{40}} = \frac{a_{40}}{a_{40}} = 1 \qquad (85)$$

$$(V) \quad F_b \cdot x_{30}^{-1} \cdot x_{40}^{-1} = \frac{q_{50}}{a_{50}} = \frac{a_{50}}{a_{50}} = 1$$

$$(VI) \quad x_{10} \cdot x_{40}^{-1} = \frac{q_{60}}{a_{60}} = \frac{a_{60}}{a_{60}} = 1$$

From I and IV of the equation it follows that:

$$x_{10} = \frac{Q_0 \cdot q_{10}}{T_{13} \cdot F_a}, \qquad (86)$$

VI according to the equation it follows that:

$$x_{40} = x_{10}, \qquad (87)$$

From equation IV will be:

$$x_{20} = \frac{F_a}{x_{10}}, \qquad (88)$$

Also, from the equation V will be:

$$x_{30} = \frac{F_b}{x_{40}} = \frac{F_b}{x_{10}}, \qquad (89)$$

The equations of system (II), (III), (VI), which at present are not used, can be used to control the results obtained with respect to all of the system equation, must be satisfied.

For example, the observed arc welding beam bracket, which are made of carbon structural steel (0.25% C), calculated constants:

- The capacity of the welding $Q_z = 0{,}05 \frac{cm^3}{s}$
- The price of basic material $T_4 = 1{,}4 \frac{CENT}{cm^3}$
- The price of electrode material $T_3 = 5{,}7 \frac{CENT}{cm^3}$
- The cost of welding device $T_{11} = 0{,}65 \frac{CENT}{s}$

Allowable stress of the base material tensile $\pm_d = 10000 \frac{N}{cm^2}$

Allowable stress of the base metal shear $^2{}_d = 5000 \frac{N}{cm^2}$

- Maximum power load beam $F = 20000\, N$
- Free length of the beam $L = 20\, cm$

Preparation costs $T_p = 73{,}9\ CENT$

Constant value to (61) will be:

$$T_{13} = \frac{T_{11}}{Q_z} + T_3 = \frac{0{,}75}{0{,}05} + 6{,}5 = 18{,}7\ \frac{CENT}{cm^3}$$

$$F_a = \frac{F}{\sqrt{2} \cdot {}^2{}_d} = \frac{20000}{\sqrt{2} \cdot 5000} = 2{,}828\ cm^2$$

$$F_b = \frac{F}{\pm_d} = \frac{20000}{10000} = 2\ cm^2\ \frac{CENT}{cm^2}$$

The components of the dual optimum vector to be (80), or according to (81):

$$q_{10} = \frac{1}{2 + \frac{T_{4L}}{F_a} \cdot \sqrt{\frac{F_b}{T_{13} \cdot T_4}}} = \frac{1}{2 + \frac{27{,}8}{2{,}828} \cdot \sqrt{\frac{2}{18{,}7 \cdot 1{,}4}}} = 0{,}2115$$

$$q_{20} = q_{10} = 0{,}2115$$

$$q_{30} = 1 - 2 \cdot q_{10} = 1 - 2 \cdot 0{,}2115 = 0{,}577$$

$$q_{40} = 2 \cdot q_{10} = 2 \cdot 0{,}2115 = 0{,}423$$

$$q_{50} = 1 - q_{10} = 1 - 0{,}2115 = 0{,}7885 \qquad (90)$$

$$q_{60} = 0$$

The optimum dual vector of (X) will be:

$$\vec{q}_0 = (q_{10}; q_{20}; q_{30}; q_{40}; q_{50}; q_{60}) = (0,2115;\ 0,2115;\ 0,577;\ 0,423;\ 0,7885;\ 0) \quad (91)$$

The optimal values of the dual function Q_0 to (69) will be:

$$Q(q_0) = \left(\frac{T_{13}}{q_1}\right)^{q_{10}} \cdot \left(\frac{T_4}{q_2}\right)^{q_{20}} \cdot \left(\frac{T_{4L}}{q_3}\right)^{q_{30}} \cdot F_a^{q_{40}} \cdot F_b^{q_{50}} \cdot 1^{q_{60}} \quad (92)$$

Substituting the values (91) to (92) shall be final:

$$Q(q_0) = \left(\frac{18,7}{0,2115}\right)^{0,2115} \cdot \left(\frac{1,4}{0,2115}\right)^{0,2115} \cdot \left(\frac{27,8}{0,577}\right)^{0,577} \cdot 2,828^{0,423} \cdot 2^{0,7885} \quad (93)$$

$$Q(q_0) = 96,4$$

where the optimal value of the dual function to function optimization:

$$F_{c0} = Q_0 = Q(q_0) = 96,4\ CENT, \quad (94)$$

On the basis of the value (86), (87) from (88), (89) are determined by the desired optimum vector $\vec{x}_0$:

$$x_{10} = \frac{Q_0 \cdot q_{10}}{T_{13} \cdot F_a} = \frac{96,4 \cdot 0,2115}{18,7 \cdot 2,828} = 0,386\ cm$$

$$x_{40} = x_{10} = 0,386\ cm$$

$$x_{20} = \frac{F_a}{x_{10}} = \frac{2,828}{0,386} = 7,326\ cm \quad (95)$$

$$x_{30} = \frac{F_b}{x_{10}} = \frac{2}{0,386} = 5,181\ cm$$

Control of the results can be performed according to the equations II, III and VI, of system (85), considering that the same are not used.

Now is the optimal primary vector completely determined:

$$\vec{q}_0 = (x_{10}; x_{20}; x_{30}; x_{40}) = (0,386; 7,367; 5,181; 0,386), \quad (96)$$

When an optimal vector (96), an optimum is achieved $F_{c0}=minF_c$ according to (62):

$$F_{c0} = T_{13} \cdot x_{10}^2 \cdot x_{20} + T_4 \cdot x_{20} \cdot x_{30} \cdot x_{40} + T_{4L} \cdot x_{30} \cdot x_{40}, \quad (97)$$

Substituting (95) into (97) will be:

$$F_c = 18,7 \cdot 0,386^2 \cdot 7,326 + 1,4 \cdot 7,326 \cdot 5,181 \cdot 0,386 + 27,8 \cdot 5,181 \cdot 0,386 = 96,4 CENT, \quad (98)$$

As might be expected, given (93).

In calculation, minimal error ocurred because of rounding of numbers (four digits).

From the above follows that the optimal values of the dimensions of the welded joint observed:

$$h_0 = x_{10} = 3,86\ mm,\ l_0 = x_{20} = 73,26\ mm,$$

$$t_0 = x_{30} = 51,81\ mm,\ b_0 = x_{40} = 3,86\ mm.$$

One can easily show that all the boundary conditions (60) are fully met.

3.2. A more complex example

As a more complex problem let's take the same example of the picture 4, with the difference that we will introduce two new constraints:

1. In view of the specific construction reasons there is a limit of geometric measure

$$t \ge b, \quad (99)$$

2. Restrictions pertaining to minimum width dimension $h_{1min}=x_{1min}$, below show that it is not possible to have technological realization of cheating:

$$h > x_{1min} \quad x_{1min} = d, \quad (100)$$

For x1min, a constant value is introduced, which may relate, for example to the minimum diameter of the applied welding electrodes, [24, 25].

Accordingly function optimization (goal) will be to (48):

$$F_c = T_{13} \cdot h^2 \cdot l + T_4 \cdot l \cdot t \cdot b + T_4 \cdot l \cdot t \cdot b, \quad (101)$$

The first three functions to limit (62), will also be as in the first example:

$$F_{g1} = F_a \cdot x_1^{-1} \cdot x_2^{-1} \le 1$$

$$F_{g2} = F_b \cdot x_3^{-1} \cdot x_4^{-1} \le 1, \quad (102)$$

$$F_{g3} = x_1 \cdot x_4^{-1} \le 1$$

Due to (99) and (100), the following two new constraints will be:

$$F_{g5} = x_4 \cdot x_3^{-1} \le 1 \quad F_{g4} = d \cdot x_1^{-1} \le 1, \quad (103)$$

Obviously, with this, we have maintained the earlier marks geometric size $x_1 = h$ $x_2 = l$ $x_3 = t$ $x_4 = b$.

All restrictions on the functions here have only one member, and according to chapter 2.2.

$$d_1 = q_4,\ d_2 = q_5,\ d_3 = q_6,\ d_4 = q_7,\ d_5 = q_8$$

For this case, an appropriate dual function, as the increased number of functions of limitation, will be more complex:

$$Q(q) = \left(\frac{T_{13}}{q_1}\right)^{q_1} \cdot \left(\frac{T_4}{q_2}\right)^{q_2} \cdot \left(\frac{T_{4L}}{q_3}\right)^{q_3} \cdot \left(\frac{F_a}{q_4}\right)^{q_4} \cdot \left(\frac{F_b}{q_5}\right)^{q_5} \cdot \left(\frac{1}{q_6}\right)^{q_6} \cdot \left(\frac{1}{q_7}\right)^{q_7} \cdot \left(\frac{1}{q_8}\right)^{q_8} \cdot q_4^{q_4} \cdot q_5^{q_5} \cdot q_6^{q_6} \cdot q_7^{q_7} \cdot q_8^{q_8} \quad (104)$$

From the conditions of normality and orthogonality (20,21), we obtain the system of equations for determining the optimal dual vectors:

$$\begin{aligned} & q_1 + q_2 + q_3 = 1 \\ & 2q_1 - q_4 + q_6 - q_7 = 0 \\ & q_1 + q_2 - q_4 = 0 \\ & q_2 + q_3 - q_5 - q_8 = 0 \\ & q_2 + q_3 - q_5 - q_6 + q_8 = 0 \end{aligned} \tag{105}$$

Apparently solving the system by (105) cannot act like in the first example, because it is obtained by a system of five linear equations with eight unknowns.

Size and parameters in the equation (104) will be in this example:

$$T_{13} = \frac{T_{11}}{Q_z} + T_3 = \frac{0,75}{0,05} + 6,5 = 18,7 \frac{CENT}{cm^3}$$

$$T_4 = 1,4 \frac{CENT}{cm^3}$$

$$T_{4L} = T_4 \cdot L = 1,6 \cdot 20 = 27,8 \ \frac{CENT}{cm^2}$$

$$F_a = \frac{F}{\sqrt{2} \cdot {}^2{}_d} = \frac{20000}{\sqrt{2} \cdot 5000} = 2,84 \ cm^2$$

$$F_b = \frac{F}{\pm_d} = \frac{20000}{10000} = 2 \quad cm^2 \ d = 0,3 \ cm^2$$

There is the adopted size d = 3 mm, in view of the minimum diameter of the electrode for carrying out the welding operation.

3.2.1. Analysis of the level of complexity

For the simple case when the number of equations (26) is equal to the number of unknown values (dual variables) of the system is obtained unambiguously (a) solution $\vec{q}_0 = (q_{j0})$. It is possible, however, and in the other (more complex) case, with the system (25a), (26), that the number of unknown size qj is greater than the number of available equations. Then we cannot talk about the optimal vector $\vec{q}_0$ with regard to $\dot{q}$, because $\dot{q}$ is multifacted, and it has many infinitely more respective solutions and the optimal solution $\vec{q}_0$ which achieve optimum Q_0, obtained by maximizing the dual function *Q (q)*, ie. solving the optimal task (dual task optimization) defined system (25a). When this is used in some of the analytical methods, for example, non-linear programming method of [13,15,7,12]. Here is the procedure of optimization easier, because all constraints are of linear shape.

Other (more complex) case is associated with the degree of complexity, which is defined by the equation:

$$s = r - (k' + 1), \tag{106}$$

where r - number of minimal positive polynomials and k'-size related to the number of independent variables in the above polynomial whose reality defines the rank of a matrix built by the exponents bij in the polynomial (9), [16,25,26]. For the case that s = 0, it does not solve the task of optimizing the dual function *Q (q)* but the $\vec{q}_0$ is determined unambiguously by the system (26).

For s=1 (the first example), as shown by the optimum solution $\vec{q}_0$ is obtained by maximizing the dual function *Q (q).*

Considering the above, the exponent for matrices another example, the system is determined by the equation (105) wherein the first equation of the system is not taken into account:

$$M_e = \begin{Vmatrix} 2 & 0 & 0 & -1 & 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & -1 & -1 & 0 & 1 \end{Vmatrix} \tag{107}$$

Rank of the matrix (107), using matrices and using matrix properties (operations with matrices) will be:

$$RangM_e = \begin{Vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{Vmatrix} = 4 \tag{108}$$

The degree of complexity is defined as

$$s = r - (k' + 1) = 8 - (4 + 1) = 3, \tag{109}$$

Here r is the number of minimizing polynomials, *k'*-size related to the number of independent variables in a positive polynomial (9) whose value defines the rank of the matrix formed by the exponents in the polynomial (9). Given that s > 0, the optimal dual vector $\vec{q}_0$ can be obtained from the equation system (9), since the number of these equations is less than the number of unknown values $q_j \cdot \vec{q}_0$. Then $\vec{q}_0$ is obtained by maximizing the dual function *Q (q)* using one of the analytical methods, for example, non-linear programming method [13-15,12].

3.2.2. Solving problems by using nonlinear programming

Substituting known parameters to a function (104), using an appropriate program by using method of nonlinear programming, to obtain the optimal dual component vectors:

$q_{10} = 0,212 \; q_{20} = 0,212 \; q_{30} = 0,576 \;\; q_{40} = 0,424$

$q_{50} = 0,788 \;\; q_{60} = 0 \;\;\; q_{70} = 0 \; q_{80} = 0$

That is:

$\vec{q}_0 = (0,212;\ 0,212;\ 0,576;\ 0,424;\ 0,788;\ 0;\ 0;\ 0)$

These values are obtained considering to $Q(q)_{max}$.

By entering the calculated values of the components $\vec{q}_0$ (X), the peak corresponding dual function (104) will be:

$$Q(q)_{max} = max\, Q(q) = Q_0 =$$
$$= Q(q_{10}; q_{20}; q_{30}; q_{40}; q_{50}; q_{60}; q_{70}; q_{80}) = 96,6 \;\; CENT$$

Thus the minimum value of function optimization will be:

$$F_{c0} = min\, F_c = max\, Q(q) = Q(q_0) = Q_0 = 96,6 \;\; CENT$$

Basis $F_{c0} = 96,6$ calculated from equation (97) components of the optimal vector according to equation (37):

$$\begin{aligned}
&T_{13} \cdot x_{10}^2 \cdot x_{20} = Q_0 \cdot q_{10} \\
&T_4 \cdot x_{20} \cdot x_{30} \cdot x_{40} = Q_0 \cdot q_{20} \\
&T_{42} \cdot x_{30} \cdot x_{40} = Q_0 \cdot q_{30} \\
&F_a \cdot x_{10}^{-1} \cdot x_{20}^{-1} = \frac{q_{40}}{a_{40}} = \frac{q_{40}}{q_{40}} = 1 \\
&F_b \cdot x_{30}^{-1} \cdot x_{40}^{-1} = \frac{q_{50}}{a_{50}} = \frac{q_{50}}{q_{50}} = 1 \\
&x_{10} \cdot x_{40}^{-1} = \frac{q_{60}}{a_{60}} = \frac{q_{60}}{q_{60}} = 1 \\
&d \cdot x_{10}^{-1} = \frac{q_{70}}{a_{40}} = \frac{q_{70}}{q_{70}} = 1 \\
&x_{40} \cdot x_{30}^{-1} = \frac{q_{80}}{a_{50}} = \frac{q_{80}}{q_{80}} = 1
\end{aligned} \qquad (110)$$

The resulting system of linear equations (110) is solved relatively easily.

From second and third equations of the system (110) that the

$$T_4 \cdot x_{20} \cdot \frac{Q_0 \cdot q_{30}}{T_{42}} = Q_0 \cdot q_{20}$$

From here it is

$$x_{20} = \frac{q_{20} \cdot T_{4l}}{T_4 \cdot q_{30}} = \frac{0,212 \cdot 27,8}{1,4 \cdot 0,576} = 7,361$$

Now, from the first equation of the system (110) that follows:

$$x_{10} = \sqrt{\frac{Q_0 \cdot q_{10}}{T_{13} \cdot x_{20}}} = \sqrt{\frac{96,6 \cdot 0,212}{18,7 \cdot 7,361}} = 0,3858$$

The fourth equation of system can be used to check the results: $F_a = x_{10} \cdot x_{20}$

Then from third equation we have:

$$x_{30} = \sqrt{\frac{Q_0 \cdot q_{30}}{T_{4L}}} = \sqrt{\frac{96,6 \cdot 0,576}{27,8}} = x_{40} = 1,41$$

From this equation it follows that $x_{30} = x_{40}$

The second equation of system may also be used to check the results, given that sizes are known in the equations. The same applies to the fifth equation system,

$$F_b = x_{30} \cdot x_{40} = 2$$

Thus the optimal size for a more complicated case of optimization will be:

$x_{10} = h_0 = 3,858\, mm$, $x_{20} = l_0 = 73,61\, mm$,

$x_{30} = t_0 = 14,10\, mm$, $x_{40} = b_0 = 14,10\, mm$

4. CONCLUSION

A method of programming is displayed in a geometrical operation, used principally in the production of various technologies. It is shown that the method under certain circumstances, is used in the field of design. Special methods efficiency is achieved when the associated technology and construction resistance are shown in the examples.

Many of the functions encountered in practice, certain mathematical transformations, can be reduced to positive polynomials and applied to the present model.

The model presented in the paper through the course of the algorithm can be considered as a more general and can be applied in many areas of design where it can be taken into account within technologies, while it is possible to apply various technical and economic criteria in optimization. All amounts to structural and technological solutions in the process of establishing the optimal project to determine the best possible. Limit function can be different both in number and shape.

Application of geometric programming is possible with different functions of optimization and constraints as linear and nonlinear. Complex problems are present in this system of linear equations that are relatively easy to solve , which is an advantage compared to other methods (for example, simplex method, and a gradient). The solution is always obtained directly without optimal search area. Special attention when applying the method of geometric programming should be processed when the limit function contains more than one member. Then the appropriate member of the effectiveness of a dual function also includes more members.

In most problems, in the end here occur more equations than necessary. This allows you to monitor and control the results with respect to all equations of the system that must be met. Also, control can be exercised towards equality *minF*$_c$ = *maxQ.*

Like any method of optimization and geometric programming method has its drawbacks.

The method can not be applied to cases where the optimization function and constraints are positive polynomials (when it appears in the polynomial minus sign) . It should be noted that the technical practices in such cases are generally rare.

Finally, it should be pointed out that modern optimization methods for efficient implementation require multidisciplinary knowledge required of different fields: technology, design,construction, economics, mathematical analysis, mathematical programming. These are probably the main reasons why we are in technical practice insufficiently engaged it cannot as well be justified.

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IZVOD

OPTIMIZACIJA DIMENZIJA OPTEREĆENOG ZAVARENOG SKLOPA METODOM GEOMETRIJSKOG PROGRAMIRANJA

U radu je na primeru jednog karakterističnog opterećenog zavarenog sklopa izvršena optimizacija njegovih dimenzija sa aspekta troškova zavarivanja. Pri ovome, postavljen je matematički model optimizacije sa finkcijama ograničenja koje pri projektovanju moraju uzeti u obzir tehnolog i konstruktor.

Za rešavanje postavljenog problema, predložen je metod geometrijskog programiranja koji je detaljno razrađen u radu u obliku algoritma pogodnog za primenu.Na taj način optimizacioni ili primarni zadatak, sveo se na dualni zadatak preko odgovarajuće funkcije, koji se znatno lakše rešava.

Metod je ilustrovan na jednom računskom praktičnom primeru sa različitim brojem funkcija ograničenja. Pokazano je da se za slučaj manjeg stepena složenosti do rešenja može doći maksimizacijom odgovarajuće dualne funkcije primenom matematičke analize. Za slučaj većeg stepena složenosti, neophodna je primena neke od metoda nelinearnog programiranja. U ovom slučaju rešenje problema je pojednostavljeno zbog svođenja linerane jednačine.

Ključne riječi: *zavarene strukture po abecednom redosledu, matematički model optimizacije, funkcija troškova, karakteristična ograničenja, geometrijsko programiranje, pozitivni polinomi, dvostruka funkcija.*

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