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Adomian decompositionmethod for modelling the growth of FeB/Fe_2B layer in boronizing process

ABSTRACT

The main objective of this paper is to explore the practical implementation of the Adomian decomposition method (ADM) in effectively solving the system of equationsgoverning boron diffusion during the boronizing process. This study uses ADM to investigate the kinetics of the boronizing process, assess the influence of various parameters on the growth of the layer thickness, and determine the boron concentration in FeB and Fe_2B phases. To validate the simulation results, data obtained from the literature were utilized. Overall, this research contributes to understanding the boronizing process and demonstrates the effectiveness of ADM as a mathematical tool for solving complex diffusion equations.

Keywords:*Adomian decomposition method, Model, Diffusion, Boronizing, kinetic,simulation*

1. INTRODUCTION

Different treatments are used to improve steel surfaces' physical, chemical, and mechanical properties [1]. Iron-based coatings have recently attracted attention for their mechanical properties [1,2], resistance to friction, and corrosion resistance. Compared to ceramics or traditional ceramic and cermet materials [3], iron-based materials offer affordability, reduced strategic importance, and can be economically manufactured using various thermal methods, making fabrication and processing simpler [1,4].

In boronizing, the modification of the surface's chemical composition is based on diffusing the boron atoms into the material. The reaction leads to the formation of a single or bilayer boride [5], which depends on the material's chemical composition and treatment process [6]. The single boride layer is preferable in industrial applications. Boronizing is frequently employed to enhance resistance against abrasion, corrosion, wear, and oxidation [1,2].It finds applications across diverse industries, including petroleum and gas refining, chemical mining, automotive, agriculture, casting, textile extrusion, and injection molding [7].

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Iron boride encompasses a range of inorganic compounds denoted by the formula Fe_xB_y, with FeB and Fe_2B being the stable and primary forms [8]. Certain iron borides exhibit advantageous traits like magnetism, electrical conductivity, corrosion resistance, and remarkable hardness. Combining ceramic attributes such as high hardness with metallic properties like thermal and electrical conductivity, iron borides offer a versatile range of properties [9].FeB exhibits greater hardness compared to Fe_2B, yet it is more prone to brittleness and fractures upon impact. Boronizing is a versatile technique applicable to numerous metals, including all ferrous alloys [6,10], as well as titanium, nickel, cobalt alloys, and refractory metals such as chromium, molybdenum, vanadium, niobium, tantalum, tungsten, and zirconium. Certain non-metals like cemented carbides can also undergo boronizing [11]. However, copper is an exception, as its atoms act as a barrier hindering the diffusion of boron atoms [12]. The speed of boronizing increases with higher substrate purity, i.e., fewer alloying elements present in the substrate [1,5], with alloying elements also influencing the thickness and structure of the resulting boride layer.

During the boronizing process, the surface layer of the material becomes saturated with boron, conducted in either a solid, liquid, or gaseous medium [13]. For steel, this process typically occurs within temperatures ranging from 1113 to 1323 K for durations of up to 10 hours, yielding FeB and Fe_2B borides with a needle-shaped

structure and hardness levels reaching up to 2000 HV [14,15].

The modeling and simulation of the growth kinetics of boride layers produced by this process allow for the optimization of the operating conditions of this treatment [16]. Kinetic models are developed to predict the thickness of the boride layer formed as a function of two critical parameters: temperature and treatment time [17].The main goal of applying these models is to predict the thicknesses of borided layers [18]. Most of these models are based on the resolution of Fick's equations. They are applied either to the study of the kinetic growth of the Fe_2B monolayer or to the bilayer (FeB+Fe_2B)[17]. Indeed, these diffusion models assume that the formation of boride layers obeys a parabolic-type law.

Brakman et al. [19] tookinto account the difference in specific volumes between FeB and Fe_2B layers, with boron concentration profiles that are linear through the FeB and Fe_2B layers. Another kinetic model was proposed by Campos et al. [20] for the growth kinetics of Fe_2B layers obtained on the surface of high-alloy steel. This kinetic approach is based on the conservation of mass equation at the interface (Fe_2B/substrate), where a linear-type distribution gives the boron diffusion profile. Keddam et al. [17] suggested a diffusion model for the bilayer (FeB+Fe_2B) formed on Armco iron by the powder method. In this model, they adopted a nonlinear boron concentration profile in each layer, taking into account the diffusion of boron in the substrate.

Experimental and artificial intelligence approaches [21] also allow measurements of the wear behavior.Bindal etal.[22] used empirical techniques to calculate the boron diffusion coefficient, Keddam et al. [23] simulated the kinetics of the boronizingprocess using the integral approach, Dybkov et al. [8,24] modeled the bilayer configuration in different processes, Mebarek et al.[25,26] developed an artificial intelligence model based on RNN and in another work proposed an FNN approach to simulate the growth of the boride layer. The Ls-SVM model for predicting the layer thickness was used by Mebarek et al.[27], and to evaluate and estimate the effect of different parameters, [28] used the fuzzy system.

Numerical computation has become a powerful tool for scientific research and is essential for engineering development. The development of approximate solution methods for mathematical and physical problems, along with powerful computers, has made it possible to simulate physical phenomena. This type of development relies on integrating traditional disciplines: applied mathematics, computer science, and engineering sciences.

Numerous methods are available to solve the diffusion model of boron in steel. In this paper, we focus on applying the Adomian Decomposition Method (ADM) to solve the Dybkov equations within the context of the boronizing process. The main objective of this research is to develop a robust and efficient computational model that accurately simulates the boron diffusion phenomenon through the surface of ferrous alloys. This study enhances the predictive capabilities of diffusion models in the boronizing treatment of steel by employing the Dybkov equations and solutions derived from the ADM method

2. DYBKOV MODEL

The diffusion model is based on Dybkov's equations [8]. It is used to study the kinetics of biphasic layer formation (FeB, Fe_2B) on steel subjected to boronizing via powder technique. Fig.1 illustrates a schematic representation of the boron concentration profiles within the FeB and Fe_2B layers.

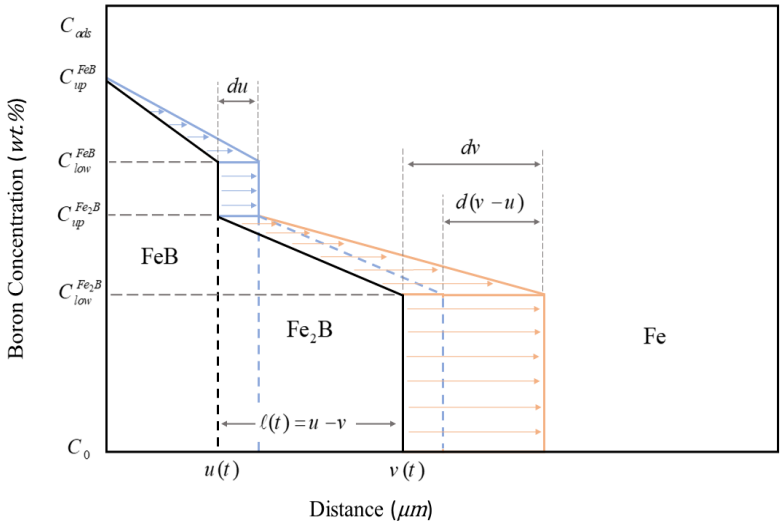

Figure 1.Schematic profile of boron concentration along the FeB and Fe_2B layers [29]

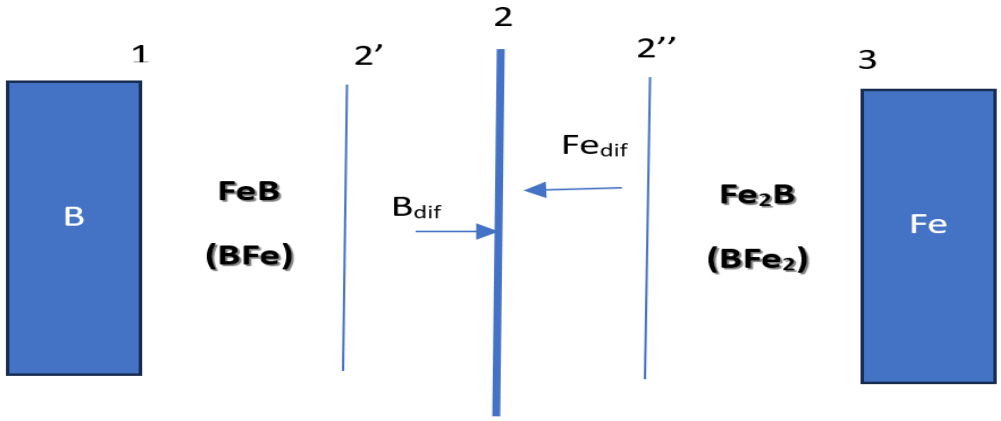

Figure 2.Schematic diagram illustrating the growth process of FeB and Fe₂B layers under diffusion-controlled conditions

The Dybkov model [8] is extensively employed to depict the growth kinetics of boronized layers on steel, accounting for the influence of boride incubation periods. According to this model, the progression of boride layers is governed by the diffusion process. Entailing the diffusion of boron atoms into the FeB layer, followed by their reaction with the Fe₂B compound. Likewise, the expansion of the Fe₂B layer is governed by the diffusion of iron atoms into its structure, followed by their reaction with the FeB compound.

The growth kinetics of layers of chemical compounds is commonly modeled using a parabolic equation of the form:

$$u = \sqrt{2k_1 t} \qquad (1)$$

Where

u - represents the thickness of the layer

k_1 - is the growth rate constant of the layer

t - is the time

This equation (1) is derived from Fick's laws of diffusion. The parabolic equations have been shown to provide a satisfactory fit to the experimental data for sufficiently thick layers in the order of micrometers, particularly for iron borides. However, for a more accurate description of the growth kinetics of FeB and Fe₂B layers, equations (2) and (3) are given.

$$\frac{dx}{dt} = \frac{a}{1+\frac{a.x}{b}} + \frac{a'}{1+\frac{a'}{b'}x} - \frac{rg}{p}\frac{c}{1+\frac{c.y}{d}} \qquad (2)$$

$$\frac{dy}{dt} = \frac{c}{1+\frac{c.y}{d}} + \frac{c'}{1+\frac{c'}{b'}} - \frac{q}{sg}\frac{a'}{1+\frac{a'x}{b'}} \qquad (3)$$

Where

x: is the thickness of the outer layer of FeB

x: is the thickness of the inner layer of Fe₂B

$a = k_0 Fe_1$ and $a' = k_0 B_1$ are the chemical growth rate constants of the FeB layer at the expense of the diffusion of iron and boron atoms, respectively

$b = k_1 Fe_1$ and $b' = k_1 B_1$ are the diffusional (physical) growth rate constants of the FeB layer at the expense of the diffusion of iron and boron atoms, respectively

$c = k_0 Fe_2$ and $c' = k_0 B_2$ the chemical growth rate constants of the Fe₂B layer at the expense of the diffusion of iron and boron atoms, respectively;

$d = k_1 Fe_2$ and $d' = k_1 B_2$ are the diffusional (physical) growth rate constants of the Fe₂B layer at the expense of the diffusion of iron and boron atoms, respectively

g is the ratio of the molar volumes of the compounds FeB and Fe₂B

$p = q = r = 1$ and $s = 2$ factors of the chemical formulas of FeB (written as BFe) and Fe₂B(BFe₂)

The value of g is determined from the density of the compounds *FeB* $\rho_1 = 6{,}70x10^3$ kgm^{-3} and *Fe₂B* $\rho_2 = 7.34x10^3$ kgm^{-3}[30] $M_1 = 66{,}65$ $gmol^{-1}$ and $M_2 = 122.49$ $g\,mol^{-1}$; $g = M_1\rho_2/M_2\rho_1 = 0.60$.

The set of equations (2) and (3) characterizes the stage of reaction-controlled growth in boride layers, wherein the rapid diffusion rates of iron and boron atoms through the layer render their influence on overall layer formation rates negligible compared to the subsequent chemical transformations occurring at phase interfaces. As a result, the rate is mainly governed by chemical processes alone (referred to as chemical control) due to the plentiful scattering of atoms of both types facilitating layer growth. However, it is essential to

note that the occurrence and growth of FeB and Fe_2B layers at the interface are not necessarily simultaneous. The Dybkov model provides a means to study the kinetics involved in the formation of FeB and Fe_2B layers. The changes over time in the thickness of these layers can be described using the following system of ordinary differential equations:

$$\frac{du(t)}{dt} = \frac{D_{FeB}}{u(t)} - \frac{rg}{p}\frac{D_{Fe_2B}}{\left(v(t)-u(t)\right)} \quad (4)$$

$$\frac{d[v(t)-u(t)]}{dt} = \frac{D_{Fe_2B}}{v(t)-u(t)} + \frac{q}{sg}\frac{D_{FeB}}{u(t)} \quad (5)$$

With *u(t)* and *v(t),* the thicknesses of FeB and Fe_2B layers. The constant g depends on the molar volume of FeB and Fe_2B phases, as obtained from the stoichiometric coefficients for phases FeB and Fe_2B.

The two growth parameters k_{FeB} and k_{Fe_2B}, which are provided by equations (6, 7) [8], can be obtained from the fitting of experimental data [19]. From the preceding system, the expressions of the diffusion coefficients are given as follows:

$$D_{FeB} = \frac{0.5\,k_{FeB}}{\left(p-\frac{rq}{s}\right)}\left[(p-rg)k_{FeB} + rgk_{Fe_2B}\right] \quad (6)$$

$$D_{Fe_2B} = \frac{0.5\left(K_{Fe_2B}-k_{FeB}\right)}{\left(1-\frac{rq}{sp}\right)\left[k+\left(\frac{q}{sg}-1\right)D_{FeB}\right]} \quad (7)$$

3. ADOMIAN DECOMPOSITION METHOD

The Adomian decomposition method has been applied to solve various types of differential and integral equations in mathematics, physics, biology, and chemistry, and many research papers have been published to demonstrate its effectiveness.

The Adomian decomposition method provides a solution expressed as an infinite series of terms, offering flexibility for reevaluation. This approach demonstrates effectiveness across various types of equations, including linear and nonlinear differential equations, integral equations, and Integro-differential equations. George Adomian introduced this method in his publications [31], characterizing it essentially as a power series technique. Widely recognized as a semi-analytical approach for solving differential equations, its convergence has been validated through prior research [32].

Previous research has shown that the Adomian decomposition method is helpful in solving inverse problems of differential equations [33]. In the current section, we extend this approach to tackle various nonlinear functional equations, including but not limited to differential, integral, integro-differential, and algebraic equations.

The Adomian decomposition method consists of decomposing the unknown function $y(x)$ of any equation into a sum of an infinite number of components defined by the decomposition series.

$$y(x) = \sum_{n=0}^{n} y_n(x) \quad (8)$$

Generally, the nonlinear differential equation is given as follows:

$$Ly + Ny + Ry = Q \quad (9)$$

Where

L : invertible linear operator

Ly : a linear term with a higher-order derivative

N : the nonlinear operator

Ny :represents the nonlinear terms

R : a linear differential operator of order less than L

Ry :remaining parts of the equation under consideration

Q: the specified bounded function

Once the inverse operator L^{-1} is applied to Eq. 9, we obtained the following equation:

$$y = c - L^{-1}Q - L^{-1}Ny - L^{-1}Ry \quad (10)$$

Where

c is the parameter resulting from the integrations such that $Lc = 0$, for the case of a first-order ODE, c is the constant of integration.

For initial value problems, we conveniently

defined L^{-1} for$L = \frac{d^n}{dx^n}$as the n-fold definite integration from 0 to x.

By employing the ADM method, wherein the solution ofEq. 9 is represented by Eq. 14 as an infinite series, and by selecting the initial solution component as:

$$Ly + Ny + Ry = Q \quad (11)$$

The solution is given by the recursive relationship given by the following system:

$$y = c\text{-}L^{-1}Q\text{-}L^{-1}Ny\text{-}L^{-1}Ry \quad (12)$$

The nonlinear terms as $Ny = \sum_{n=0}^{+\infty} A_n$ is decomposed by the ADM

where:

$A_n$represent the Adomian polynomials given by the following expression:

$$A_n(y_0, y_1, \dots y_n)\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N\left(\sum_{i=0}^{+\infty}\lambda^i y_i\right)\right]_{\lambda=0} \quad (13)$$

Using the recursive relationship given by the precedent system (12), we can construct the solution y as:

$$y(x) = \lim_{i\to+\infty}\sum_{i=0}^{i=n} y_n(x) \quad (14)$$

Many researchers have guaranteed the convergence of the ADM and the series of the Adomian polynomials [32,34].We not thatADM can converge to the exact solution more efficiently under certain conditions, especially for linear and non-linear differential equations. Generally ADM provides a series solution that can offer analytical insight into the problem, which is often not directly available with numerical methods.

4. SOLUTION OF THE DYBKOV SYSTEM BY ADM

In this study, we adapted the model calculation of Fatoorehchi et al. [35], which was applied to describe the growth dynamics of Al_3Mg_2 and $Al_{12}Mg_{17}$ layers. However, in this case, we applied the ADM to modeling the boronizing process.

From Eq. 4 and Eq. 5, we replace u in the equation system by x and $(v-u)$ by y. We get the growth kinetics of boride layers at the diffusional stage of their formation described by a system of two differential equations:

$$\frac{dx}{dt} = \frac{D_{FeB}}{x} - \frac{rg}{p}\frac{D_{Fe_2B}}{y} \quad (15)$$

$$\frac{dy}{dt} = \frac{D_{Fe_2B}}{y} - \frac{q}{sg}\frac{D_{FeB}}{x} \quad (16)$$

where the solution components are calculated as

$$\begin{cases} y_0 = \alpha + \beta t \\ y_{i+1} = -\boldsymbol{D_{Fe_2B}}\int_0^t dt\int_0^t A_i\,dt' + \frac{s^2g^2}{q^2\boldsymbol{D_{FeB}}}\int_0^t dt\int_0^t B_i dt' - \frac{sg^2r\boldsymbol{D_{Fe_2B}}}{qp\boldsymbol{D_{FeB}}}\int_0^t dt\int_0^t C_i dt' \end{cases} \quad (21)$$

For$i \geq 0$. Two parameters, α,andβ, have to be determined by the initial conditions of theproblem, i.e.

$$\alpha = y(0) \qquad \beta = \frac{dy}{dt}(0) \quad (22)$$

We apply the ADM method to solve the system (15) and (16) given by Dybkovmodel. This system describes the diffusion of boron in the treated steel.

With the use of the precedent system, the expression of x is as follows:

$$x = \frac{qD_{FeB}}{sg}\left(\frac{D_{Fe_2B}}{y} - \frac{dy}{dt}\right)^{-1} \quad (17)$$

Withthe derivative of x, we obtained the equation given by:

$$\frac{dx}{dt} = \frac{qD_{FeB}}{sg}\frac{\left(\frac{D_{Fe_2B}dy}{y^2\ dt} + \frac{d^2y}{dt^2}\right)}{\left(\frac{D_{Fe_2B}}{y} - \frac{dy}{dt}\right)^2} \quad (18)$$

We replace Equations (17) and (18) into Equation (15) to obtain the second-order ODE given by Equation (19):

$$\frac{d^2y}{dt^2} = -\frac{D_{Fe_2B}}{y^2}\frac{dy}{dt} + \frac{s^2g^2}{q^2D_{FeB}}\left(\frac{D_{Fe_2B}}{y} - \frac{dy}{dt}\right)^3 - \frac{sg^2rD_{Fe_2B}}{qpD_{FeB}}\frac{1}{y}\left(\frac{D_{Fe_2B}}{y} - \frac{dy}{dt}\right)^2 \quad (19)$$

The ODE given in Eq. 19 can be solved by the ADM, and its solution is given as:

$$y = \sum_{i=0}^{\infty} y_i \quad (20)$$

In (19) we use the following operator

$$L(.) = \frac{d^2}{(dt^2)(.)}$$

and we note that

$$L^{-1}(.) = \int_0^t dt\int_0^t dt'(.)$$

In turn, A_i , B_i and C_i are the Adomian polynomials such that

$$\sum_{i=0}^{+\infty} A_i = \frac{1}{y^2}\frac{dy}{dt} \ , \ \sum_{i=0}^{+\infty} B_i = \left(\frac{D_{Fe_2B}}{y} - \frac{dy}{dt}\right)^3 , \sum_{i=0}^{+\infty} C_i = \left(\frac{D_{Fe_2B}}{y} - \frac{dy}{dt}\right)^2 \quad (23)$$

After evaluating the function y, calculating the function x using Eq. (15) is simple.

5. EXPERIMENTAL PROCEDURE

In order to validate the simulation, experimental data carried out by Brakman et al. [19] using the boronizing technique on pure iron powders were used, and the chemical composition of pure iron is presented in Table 1.

Table 1. Chemical composition of used pure iron (mass%) [19].

Elements	C	Mn	Ni
%(wt)	0.005	0.0013	0.004

Boronizingwith solid substances has numerous industrial applications due to its cleanliness and simplicity. The technique involves packing the parts in steel boxes filled with boronizingpowder (amorphous boron or ferroboron) and introducing them into muffle furnaces. It is the most widely used technique in the industry.

FeB and Fe_2B borides have an acicular shape and are oriented perpendicularly to the sample's surface. In practice, it is not easy to experimentally measure the thickness of the boride layer due to the nature and structure of the boride/substrate interface. The thickness determined experimentally for biphasic borided layers is the average of 10 measurements made at different locations on the sample [19]. Consequently, experimental data regarding the thickness of the FeB and Fe_2B layers are illustrated in Table 2.From the experimental data presented in Table 2 of layerthickness,we calculated the values of the growth rate constants, given in Table 3.

Table 2.Experimental data of FeB and Fe_2B layer thickness [19]

Time (h) / Temperature (K)	1	1.25	2.5	4	4.25	6	8	8.5
	FeB (µm)							
1023				6.96		8.52	9.842	
1073	6.84				14.1			19.94
1123		13.15	16.59	21.52				
	Fe_2B (µm)							
1023				37.08		45.41	52.44	
1073	25.5				52.56			75.34
1123		42.13	59.58	75.34				

6RESULTS AND DISCUSSIONS

The ADM is applied to describe the growth dynamics of FeB and Fe_2Blayers, denoted by xand y, respectively. The five numerical values of the Adomian coefficients A_i, B_i, and C_i used in the calculations are taken from Fatoorehchia et al. [35].

Theresults are compared with the experimental data from [19]. $D_B^{Fe_2B}$ and D_B^{FeB} are the growth rate constantsfor Fe_2B and FeBgiven by the following expressions [17]:

$$D_B^{FeB} = 1.28\times10^{-5}\exp(-\frac{175\times10^3}{RT})$$

$$D_B^{Fe_2B} = 6.3\times10^{-6}\exp(-\frac{157\times10^3}{RT})$$

Where, *R=8.32 j/mol k*

To simulate the boronizing kinetics using a simple model, we set the boron concentration at the surface to 27.26 wt%.Additionally, we assumed that the concentration of boron at the interface (FeB/Fe_2B) is 16.23 wt%. Specifically, for the Fe_2B/Fe interface,the boron concentration is 8.83wt%. The initial concentration of boron in the substrate is $35x10^{-4}$ wt%.

Using a biphasic boride layer kinetic model and the boron diffusivity data, a simulation code was developed to calculate the thickness of the Fe_2B and FeB layers and predict the boron concentration.

The ADM approach is used to simulate the Dybkov model, utilizing the parameters illustrated

in this simulation. The input data for the simulation calculations comprises a ratio of molar volumes of Fe_2B and FeB denoted as g = 0.6. Additionally, α = $0.1012x10^{-13}$m represents the initial layer thickness y, and the initial slope of y is denoted asβ, which is equal to$5.5x10^{-15}$ m/s.

The stoichiometric coefficients used in this study are specified as (p, q, r, s) = (1, 1, 1, 2).

Table 3 depicts the comparison between the simulation and experimental data.Comparing the calculated growth rate constants with the experimental values reveals a strong agreement between the experimental data and the simulation results.

Table 3. Comparing the calculated growth rate constants with the experimental ones

Temperature (K)	Growth rate constant (μm/s)					
	FeB			Fe_2B		
	Exp [19]	Simulation		Exp [19]	Simulation	
		Simple Model [17]	ADM-Dybkov Model		Simple Model [17]	ADM-Dybkov Model
1023	0.058	0.0607	0.0635	0.3090	0.2818	0.2988
1073	0.144	0.1012	0.1123	0.4250	0.4356	0.4527
1123	0.196	0.1611	0.1722	0.6280	0.6479	0.6650

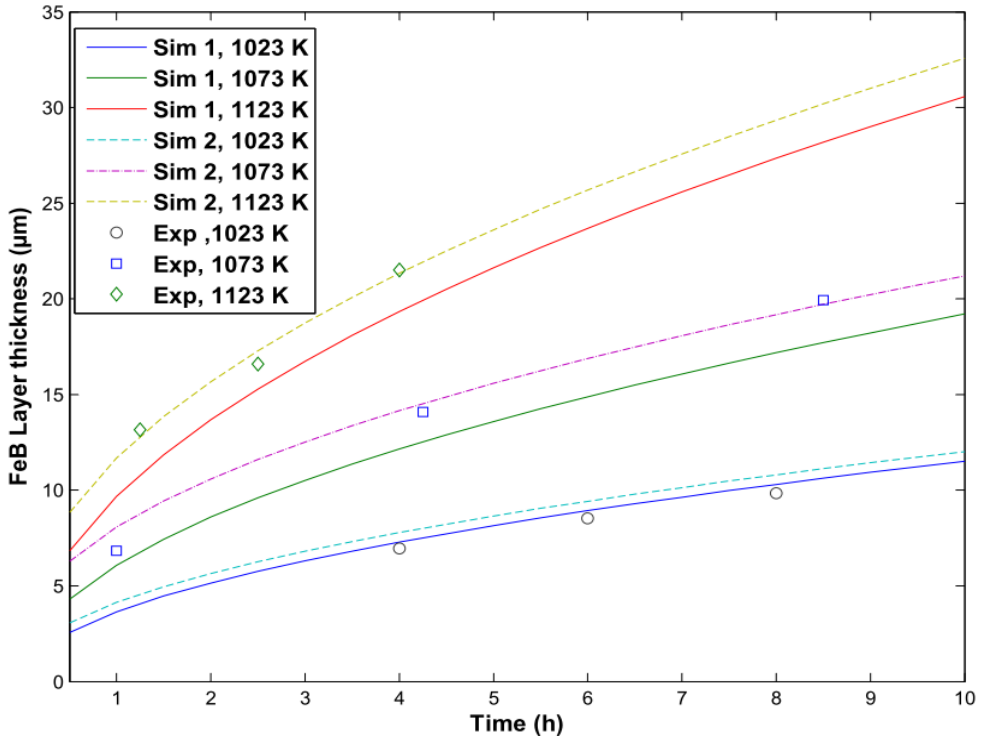

Figure 3. Comparison between Sim1 (simple model) [17], Sim2 (ADM-Dybkov model) and Experimental thicknessesof FeB layer

Fig.3 illustrates a comparative analysis between Sim1 (simple model), Sim2 (ADM-Dybkov model), and experimental thicknessesof the FeB layer. This comparison aims to evaluate the performance and accuracy of these models.

Fig.4 presents a comparative assessment involving Sim1 (simple model), Sim2 (ADM-Dybkov model), and experimental data related to the Fe_2B layer. The objective of this comparison is to assess the effectiveness and precision of these models.

This comparison between Sim1 (simple model), Sim2 (ADM-Dybkov model), and experimental data of the Fe_2B layer provides a comprehensive evaluation of the model's predictive capabilities in simulating the growth kinetics of the boronized layers on steel. The ADM-Dybkov model gives good results when compared with experimental data. The error of the ADM-Dybkov model compared to the experimental data is estimated to be 2.9 μm for the Fe_2B layer and 1.4 μm for the FeB layer.

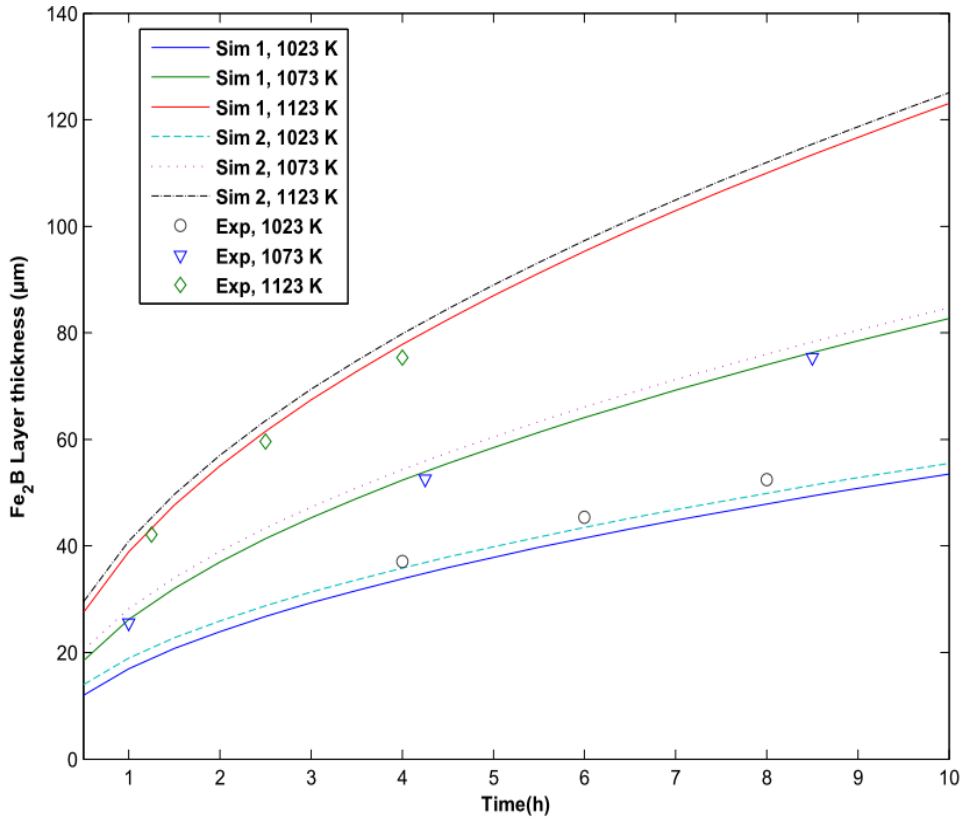

Figure 4.Comparison between Sim1 (simple model) [17], Sim 2 (ADM-Dybkov model) and Experimental data of Fe_2B layer

7. CONCLUSION

This study introduces the Adomian decomposition method (ADM) as a robust strategy for approximating solutions to nonlinear equations through an infinite series, typically converging to the precise solution. Applied to first-order differential equations, ADM exhibits convergence of series solutions to exact solutions for each problem. Utilizing ADM, the study effectively models boron diffusion in the boronizing process, demonstrating its capability to closely match experimental data, which are the thicknesses of FeB and Fe_2B layers. The investigation delves into the kinetics of boronizing, exploring how various parameters influence layer thickness growth and boron concentration across phases. Simulation results are validated using data from literature sources, underscoring ADM's effectiveness in tackling intricate diffusion equations and advancing understanding of the boronizing process.

As prospects for future work, we aim to combine ADM with other computational techniques, such as machine learning and optimization algorithms, to create hybrid models. These models could enhance predictive accuracy and provide more robust solutions for complex problems.

APPENDIX

We assume a standard form of an integro-differential equation defined by the standard form:

$$\begin{cases} y^{(n)}(x) = f(x) + \int_0^x M(x,t)y(t)dt \\ y^{(n)}(0) = b_k\,, \qquad 0 \le k \le n-1 \end{cases} \qquad (A)$$

It is natural to seek an expression for *y(x)* that will be derived from equation (A). This can be done by integrating both sides of equation (A) from 0 to *x* as many times as the order of derivative involved. Therefore, we obtain:

$$y(x) = \left[\sum_{k=0}^{n-1} \frac{1}{k!} b_k x^k\right] + L^{-1}\left(f(x)\right) + L^{-1}\left(\int_0^x M(x,t)y(t)dt\right) \qquad (B)$$

With

$$\sum_{k=0}^{n-1} \frac{1}{k!} b_k x^k$$

the solution is obtained using the initial conditions, and L^{-1} is an integration operator. Now, we can

apply the decomposition method by defining the solution y(x) of equation (B) as a series.

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \qquad (C)$$

By substituting equation *(C)* into *(B),* we obtain

$$\sum_{n=0}^{\infty} y_n(x) = \left[\sum_{k=0}^{n-1} \frac{1}{k!} b_k x^k\right] + L^{-1}\left(f(x)\right) + \cdot$$
$$+ L^{-1}\left(\int_a^x M(x,t)\left[\sum_{n=0}^{\infty} y_n(t)\right]dt\right) \qquad (D)$$

This equation can be explicitly written as:

$$\sum_{n=0}^{\infty} y_n(x) = \sum_{k=0}^{n-1} \frac{1}{k!} b_k x^k + L^{-1}\left(f(x)\right) +$$
$$+ L^{-1}\left(\int_a^x M(x,t)\left[\sum_{n=0}^{\infty} y_n(t)\right]dt\right) \qquad (E)$$

We have the solution in the form:

$$y_0(x) = \sum_{k=0}^{n-1} \frac{1}{k!} b_k x^k + L^{-1}(f(x)) \qquad (F)$$

$$y_{n+1}(x) = L^{-1}\left(\int_a^x M(x,t) y_n(t) dt\right) n \geq 0 \qquad (G)$$

8. REFERENCES

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IZVOD

ADOMIJANSKA DEKOMPOZICIONA METODA ZA MODELIRANJE RASTA FeB/Fe_2B SLOJA U BORONIZIRANJE PROCESA

Glavni cilj ovog rada je da se ispita praktična implementacija Adomianove metode dekompozicije (ADM) u efikasnom rješavanju sistemske jednadžbe koja upravlja difuzijom bora tokom procesa boriranja. Ovaj rad koristi gore pomenutu metodu za istraživanje kinetike boroniziranje procesa, procjenu utjecaja različitih parametara na rast debljine sloja i određivanje koncentracije bora u FeB i Fe_2B fazama. Za validaciju rezultata simulacije koristili smo podatke dobijene iz literature.

Ključne riječi: *Adomian metoda dekompozicije, Model, Difuzija, Boroniziranje, kinetička, simulacija*

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