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Confrontation of linear versus nonlinear approach in Fe_2B boride layer thickness predictions

ABSTRACT

Kinetic studies of boride layers focus on trying to accurately predict their thicknesses according to some variables using different approaches. In this paper, an approach that is reliant on a multilinear regression is investigated. In doing so, with an engineering perspective, temperature T and time t are used as the sole variables in predicting a boride layer thickness u. The approach uses experimental data from a boriding process performed on iron substrates of the XC38 steel. A comparison between the proposed linear model and a nonlinear one is seen afterward to scrutinize the results. That nonlinear approach is known as the diffusion model and is based on Fick's second law, where it uses more variables than the linear approach to estimate its predictions. Ultimately, the comparison elucidated that the use of a linear regression-based model can be an accurate engineering tool to identify boride layer thicknesses, but without interpolating the results outside the scope of the studied interval.

Keywords: *Multilinear regression / Linear regression / Boriding / Boride layer / Boron.*

1. INTRODUCTION

Different hardening processes have been envisaged since the revolution of industrial machining. These processes focus on surface treatments of tools and parts, enhancing wear resistance during use to further prolong lifespan. Nitriding, carbonizing, chromium plating, thermal spraying, chemical vapor deposition or physical vapor deposition coatings are some known processes. Boriding is one such process where significant advantages of mechanical properties are reached [1 - 3].

Boriding is widely spread over diverse fields like fittings, gear transmissions, milling and crushing technologies, extrusion techniques, stamping, tools, renewable energies, and others [4,5]. The process is also known as boronizing or boron coating, a thermo-chemical treatment carried out in solid, liquid or gaseous media, hence the naming of the processes. E.g., gas boriding, paste boriding, powder pack boriding process or others [1,5,6].

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The most common one is the latter, and it is done by immersing parts in a boriding agent, where there are different boron sources (B4C, amorphous boron, ferroboron), activators ($NaBF_4$, KBF_4, NH_4Cl, Na_2CO_3, NiO) and diluents (SiC, SiO_2, Si_3N_4, Al_2O_3), then sealing them in a heat resistant steel furnace [5,7]. During the process, intermetallic compounds are formed between the boron and other elements, for instance, iron, chromium, nickel, vanadium, cobalt, molybdenum, tungsten, and titanium [8], depending upon the material processed. Borided materials range from cast iron, cast steel, and most steel grades to nickel-based alloys and specific materials such as stellite, except silicon and aluminum [9].

Each boriding process is founded upon the diffusion of boron atoms at high temperatures during a time period, resulting in either a monophase layer or a multiphase one [10,11]. This particular layer has multiple valuable properties, increased resistance to wear from abrasions [12], acids [13], oxidations [14], corrosions [15], erosions [16], and adhesions by decreasing friction [17,18]. Depending on the base material and other aspects, it also increases hardness to about 1000 or 2600 HV and even higher [19,20]. All those factors expand the service life of parts by improving their degradation resistance [21].

Throughout the advancement of research and the increase of its expenditure, modeling the kinetics of that boride layer became predominantly essential. Doing so demanded the characterization of the required variables to predict the boride layer thickness with less cost and fewer experiments. Specific models were developed by researchers, and further investigations were carried out. Presenting a few, the parabolic growth law model [22], the alternative diffusion model [23], the Dybkov model [24], the integral model [25,26] and the diffusion model that is based on the second law of Fick [26-30]. All these models are nonlinear and have a common goal: to analyze the diffusion data by studying the kinetics of boride layer thicknesses.

In this study, data of a boronized XC38 steel substrate [28] is explored through an attempt to predict the growth of the Fe_2B boride layer thickness u with a linear-based model that uses engineering lenses, hence, needing the least variables possible, temperature T and time t. After getting the predicted results, they are compared with a nonlinear model presented by Mebarek *et al.* in previous research, the diffusion model [28]. This comparison allows us to see if not taking other variables, such as incubation time t_0, boron concentrations C and the diffusion coefficients D, into account can mislead or falsify predictions.

Primarily, the study begins by developing the theories that support both the nonlinear model of Mebarek *et al.* [28] and the proposed linear model. After that, calculating and illustrating the results, then comparing each model's results by assessing their errors with the experimental data. The purpose is to ascertain if using engineering lenses via a linear model can be considered in the studies of Fe_2B boride layer kinetics due to it needing fewer variables, thus, ease of use.

2. EXPERIMENTAL DATA

The linear approach can be of use from an engineering point of view and is validated through comparison with another nonlinear approach, the diffusion model, published previously by Mebarek *et al.* [27]. Both approaches use the same experiment results obtained from boronizing substrates of XC38 steel [29]. Moreover, the samples' composition is given in Table 1, and the thermochemical boriding experiments were carried out in liquid mediums composed of borax ($Na_2B_4O_7$) and 30 $wt.\%$ silicon carbide (SiC).

Table 1. Composition of the XC38 steel samples

Tabela 1. Sastav uzoraka od čelika XC38

Elements	C	Cr	Cu	Ni	Co	Si	Mn	Fe
[$wt.\%$]	≈0.38	<0.1	<0.05	≈0.045	≈0.17	≈0.34	≈0.67	balance

The experiments were performed at three different temperatures, 850, 950 and 1000 $°C$ with three treatment times, two, four and six hours.

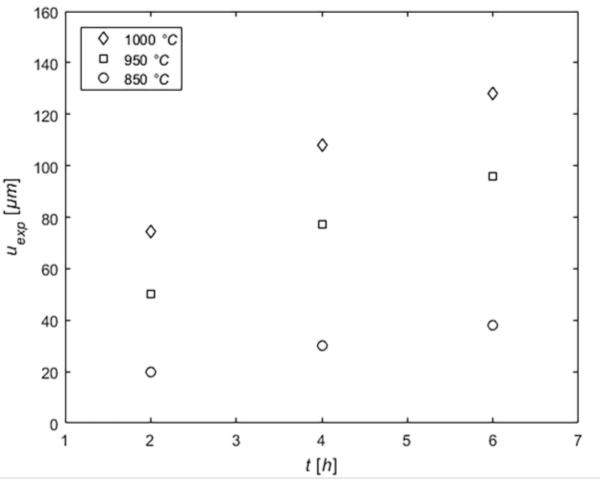

Figure 1. Boriding XC38 steel from 2 to 6 h on 850, 950 and 1000 $°C$

Slika 1. Boronizacija čelika XC38 u trajanju od 2 do 6 h na temperaturi 850, 950 i 1000 °C

By observations under an optical and electron microscope with sweeping, Fe_2B boride genesis was observed and different layer thicknesses were obtained depending on temperatures and treatment times, as illustrated in Figure 1.

3. MODELS

Diffusion model

The diffusion model is nonlinear and uses an approach presented by Mebarek *et al.* [28]. It is based on the equation of Fick used on semi-infinite media where different equations are summoned in order to characterize the growth rate constant k with which the model is established from experimental data. The boron concentration profile is illustrated in Figure 2, and described by the diffusion equation (1), the second law given by Fick.

$$\frac{\partial C_i}{\partial t} = D_i \frac{\partial^2 C_i}{\partial x^2} \quad (1)$$

As all models have assumptions, this model's assumptions are given as follows:

- The flow of boron atoms is perpendicular to the sample surface, and the interface runs parallel to the sample surface.
- The growth of the position of the interface according to time is parabolic.
- The borided layer is thin compared to the thickness of the sample.
- The porosity effect does not exist on the surface of the material.
- The borided layer Fe_2B is formed instantly and immediately covers the surface (inexistence of the incubation time t_0).
- The diffusion coefficient is constant with composition.

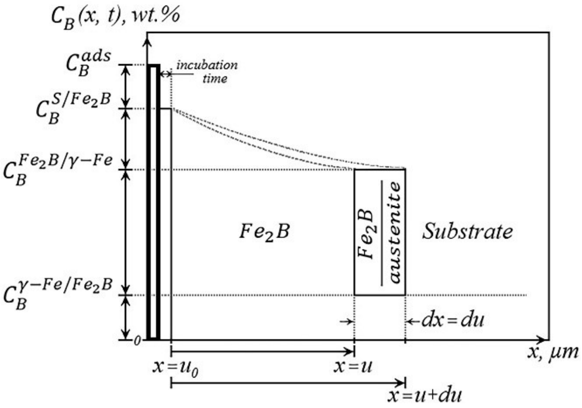

Figure 2. Boron concentration throughout the Fe_2B boride layer thickness

Slika 2. Koncentracija bora u celoj debljini sloja Fe_2B borida

The general solution of the first given equation for each phase i (Fe_2B and γ-Fe) is given as equation (2) which characterizes the general concentration equation:

$$C_i(x,t) = A_i + B_i erf\left(\frac{x}{2\sqrt{D_i t}}\right) \qquad (2)$$

where

A_i and B_i constants

erf Gaussian error function

x boride layer thickness

The boride layer thickness u is simulated via a parabolic law of time t and the growth rate constant k whose value is unknown, as given in equation (3).

$$u_{sim} = k\sqrt{t} \qquad (3)$$

The Boundary conditions of the study are:

At initial conditions and limits

$$C(x,0) = 0,$$

$$C(0,t) = C_B^{S/Fe_2B},$$

$$C(\infty,t) = 0.$$

At interface

$$C_{Fe_2B}(u_{sim},t) = C_B^{\frac{Fe_2B}{\gamma}-Fe},$$

$$C_{\gamma-Fe}(u_{sim},t) = C_B^{\gamma-Fe/Fe_2B}$$

where

C_B^{S/Fe_2B} at the sample surface

$C_B^{Fe_2B/\gamma-Fe}$ *and* $C_B^{\gamma-Fe/Fe_2B}$

in the interface of Fe_2B and γ-Fe.

Using equations (2), (3) and the boundary conditions, the concentrations of each phase i (Fe_2B and γ-Fe), are reached as follows:

$$C_{Fe_2B}(x,t) = C_B^{S/Fe_2B} + \frac{C_B^{Fe_2B/\gamma-Fe} - C_B^{S/Fe_2B}}{erf\left(\frac{k}{2\sqrt{D_{Fe_2B}}}\right)} erf\left(\frac{x}{2\sqrt{D_{Fe_2B}t}}\right) \qquad (4)$$

$$C_{\gamma-Fe}(x,t) = \frac{C_B^{\gamma-Fe/Fe_2B}}{erfc\left(\frac{k}{2\sqrt{D_{\gamma-Fe}}}\right)} erfc\left(\frac{x}{2\sqrt{D_{\gamma-Fe}t}}\right) \qquad (5)$$

The mass balance equation for the Fe_2B/γ-Fe interface is derived from the literature, equation (6):

$$W\frac{du_{sim}}{dt} = \Delta J_{x=u_{sim}} \qquad (6)$$

where

$$W = \frac{1}{2}\left(C_B^{S/Fe_2B} - C_B^{Fe_2B/\gamma-Fe}\right)$$

$$+\left(C_B^{Fe_2B/\gamma-Fe} - C_B^{\gamma-Fe/Fe_2B}\right)$$

$$\Delta J_{x=u_{sim}} = \left(J_{i_1} - J_{i_2}\right)_{x=u_{sim}}$$

$$J_i = -D_i\frac{\partial C_i(x,t)}{\partial x}$$

J_i represents the flow.

Using equations (3), (4) and (5) within equation (6) and by rearranging it, only the growth rate constant k remains as the sole non determined variable, as in equation (7):

$$f(k) = \frac{Wk}{2} + \frac{1}{\sqrt{\pi}}\left[\begin{array}{c}\left(C_B^{S/Fe_2B} - C_B^{Fe_2B/\gamma-Fe}\right)\sqrt{D_{Fe_2B}}\,\dfrac{exp\left(-\frac{k^2}{4D_{Fe_2B}}\right)}{erf\left(\frac{k}{2\sqrt{D_{Fe_2B}}}\right)} + \\ + \left(C_B^{Fe_2B/\gamma-Fe} - C_B^{\gamma-Fe/Fe_2B}\right)\sqrt{D_{\gamma-Fe}}\,\dfrac{exp\left(-\frac{k^2}{4D_{\gamma-Fe}}\right)}{erfc\left(\frac{k}{2\sqrt{D_{\gamma-Fe}}}\right)}\end{array}\right] \quad (7)$$

The rest of the parameters of equation (7) are: temperature T, treatment time t and activation energy Q found in the boron diffusivity D in each phase i and its concentration C. These parameters can be determined.

- For the Fe_2B phase, an estimation of the diffusivity of boron D in it has been attained through the Arrhenius relation for the diffusion process, equation (8):

$$u^2 = D_0 t\, exp\left(-\frac{Q}{RT}\right) \quad (8)$$

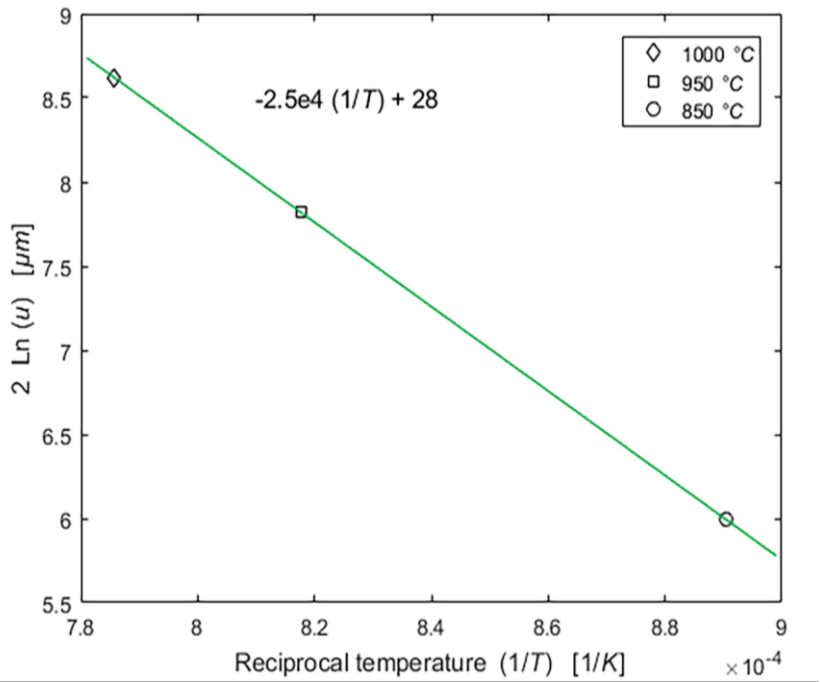

Figure 3. Reciprocal temperature dependence of the boride layer thickness formed after 2 h

Slika 3. Zavisnost debljine boridnog sloja formiranog nakon 2 h od vrednosti recipročne temperature

By linearizing the equation (8) and plotting it using 2 h of treatment time from experimental data as of Figure 3, the activation energy Q was deduced as 207.8 kJ.

Thus, a diffusivity of boron in the Fe_2B phase was attained as:

$$D_{Fe_2B} = 1.388 \cdot 10^{-4}\, exp\left(-\frac{207.8 \cdot 10^3}{RT}\right)\ [m^2 s^{-1}]$$

- For the γ-Fe phase, the diffusion coefficient found in reference [30] was used.

$$D_{\gamma-Fe} = 4.4 \cdot 10^{-8}\, exp\left(-\frac{91.51 \cdot 10^3}{RT}\right)\ [m^2 s^{-1}]$$

- As for the concentrations, they were taken from references [31] and [32].

$$C_B^{Fe_2B/\gamma-Fe} = 8.83\ wt.\%$$
$$C_B^{\gamma-Fe/Fe_2B} = 35 \cdot 10^{-4}\ wt.\%$$

Incorporating all previously determined parameters into equation (7) and through nonlinear solving methods such as Newton-Raphson's numerical method by finding its root, in other terms, solving $f(k) = 0$, a positive value of the growth rate constant k can be determined. The calculated growth rate constant k is given in Table 2.

Table 2. Evolution of the calculated growth rate constant according to temperature

Tabela 2. Zavisnost izračunate konstante brzine rasta od temperature

T [°C]	750	800	850	900	950	1000	1050
k [$\mu m\ s^{-1/2}$]	0.1481	0.2275	0.3365	0.4800	0.6672	0.9037	1.1960

After having the growth rate constant k values, the approach of Mebarek *et al.* [28] proceeds to overcome the last assumption made earlier about the incubation time t_0 by exploiting a mathematical term that simulates it (incubation time), equation (9), using the difference between the simulated boride layer thickness u_{sim} and the experimental data u_{exp}.

Table 3 gives the values gotten of that simulated incubation time.

$$t_0(t,T) = \left(\frac{u_{sim} - u_{exp}}{k}\right)^2 \quad (9)$$

Table 3. The simulated incubation time values.

Tabela 3. Simulirane vrednosti vremena inkubacije.

	T [°C]	t [h]		
		2	4	6
t_0 [s]	850	217.39	320.19	389.96
	950	65.56	14.07	6.35
	1000	5.27	0.22	25.67

After that, a unitless parameter was added to predict the boride layer thickness [28]. This parameter is identified as $B(T)$ and helps in taking the incubation time t_0 into consideration after estimating it depending on the temperature T as illustrated in Figure 4. Its equation (10) is given as:

$$B(T) = 1 - \sqrt{t_0/t} \quad (10)$$

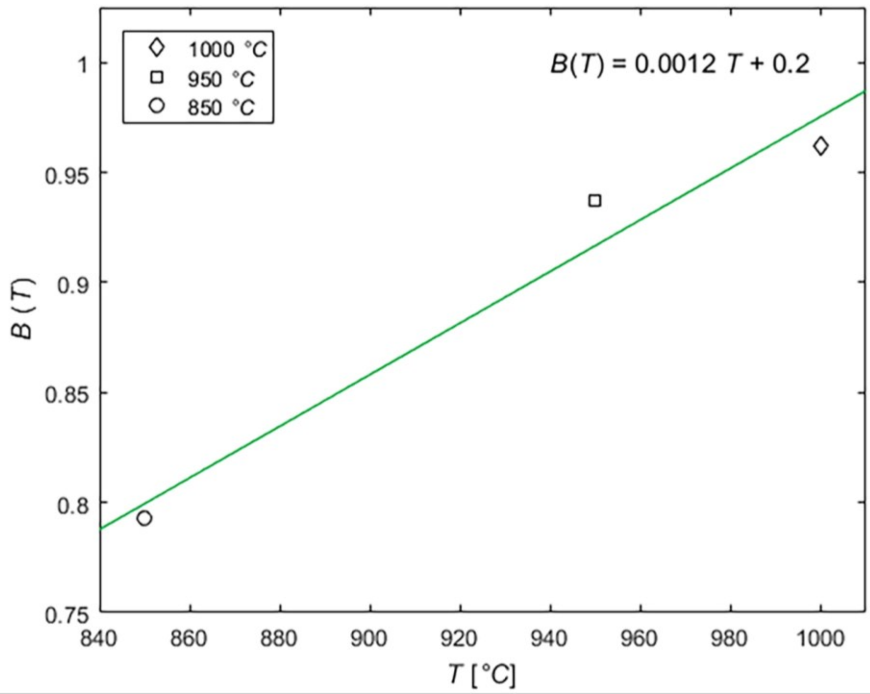

Figure 4. Evolution of parameter B in accordance with temperatures of 2 h

Slika 4. Zavisnost parametra B od temperature nakon 2 h

In the end, the equation (11) was used in this approach's predictions:

$$u_D = kB\sqrt{t} \quad (11)$$

Linear model

The proposed linear model utilizes a multilinear regression, also called multiple linear regression. Regressions are statistical techniques that employ numerous independent variables to forecast or determine the value of a dependent variable. Its general equation (12) is found in multiple literatures, such as [33]:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \varepsilon \quad (12)$$

where

y the dependent or predicted variable

$x_1\ x_2\ \ldots\ x_p$ the independent variables

$\beta_0\ \beta_1\ \beta_2\ \ldots\ \beta_p$ regression coefficients

ε the model's error term (residual)

The multilinear regression is based on certain assumptions [34, 35]:

- Dependent and independent variables must have linear relationships.
- Non-collinearity; the independent variables are not highly correlated with each other.
- The variance of the residuals is constant.
- Independence of observation.
- Multivariate normality.

In this studied case, there are two variables, temperature and time. Thus, the equation (12) becomes equation (13):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon \quad (13)$$

These two variables have a nonlinear relationship, which doesn't satisfy the earlier assumptions. Furthermore, advanced multilinear regressions can overcome this issue by adding an interaction term [35] to the equation (13) between variables with nonlinear relationships. Resulting in a model having independent variables and an interaction term that adjusts the nonlinear relationship as in equation (14):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \varepsilon \quad (14)$$

where

$x_1 x_2$ represent the interaction term

4. COMPUTATIONS

The first nonlinear approach's calculations were done in reference [28]. Hence, this section is dedicated to the second one, the proposed linear model, the advanced multilinear regression.

Equating the boride layer thickness

Utilizing the equation (14) of the multilinear regression model, our approach's equation (15) becomes as follows:

$$u_i(T_i, t_i) = \beta_0 + \beta_1 T_i + \beta_2 t_i + \beta_3 T_i t_i \quad (15)$$

where

i represents the data points

u is the predicted boride layer thickness

T and t are the independent variables of temperature and time

$\beta_0\ \beta_1\ \beta_2\ \beta_3$ are the regression coefficients

This equation (15) is applied to all nine experimental data points given in Figure 1 gives nine equations.

Forming the matrixial system

The nine equations obtained from equation (15) are rewritten in a matrixial form, equation (16), for computing purposes [36] in order to facilitate calculations by using an automated computation that calculates any data.

$$U = X\,\beta \quad (16)$$

where

$$U = [\ u_i\ ]$$

$$X = [\ 1 \quad T_i \quad t_i \quad T_i t_i\ ]$$

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

$i = 1 : n$ while n represents the number of data points

U a vector column of the data boride layer thicknesses u

X a $(n \times 4)$ matrix of the independent variables and the interaction term

β a vector column of the regression coefficients

Resolving the system

The resolution of the matrixial system gives the regression coefficients and can either be done via a computing platform or mathematically [37] with the formula below:

$$\beta = (X'X)^{-1}\,X'\,U \quad (17)$$

After resolution, the obtained regression coefficients are given in Table 4.

Table 4. The obtained regression coefficients of the linear model

Tabela 4. Dobijeni koeficijenti regresije za linearni model

Coefficients $[10^{-3}]$	β_0	β_1	β_2	β_3
	-199000	245.2381	-46875	60.7143

Establishing the linear model

In the end, through the multilinear regression, the equation (18) of the linear approach is attained and helps to determine the boride layer thickness u depending upon any given temperature T in $°C$ and time t by hours.

$$u_L(T,t) = \beta_0 + \beta_1 T + \beta_2 t + \beta_3 Tt \tag{18}$$

where

u_L represents the boride layer thickness predicted using the regression

Assessing the errors

The comparison of the results was conducted using residuals, the mean absolute error (MAE) as of equation (19), the standard error of the estimate (SEE), equation (20) and the minimal and maximal errors.

$$MAE = \frac{1}{n}\sum_{i=1}^{n} | u_i - \bar{u} | \tag{19}$$

$$SEE = \sqrt{\frac{\sum_{i=1}^{n}(u_i - \bar{u})^2}{n-2}} \tag{20}$$

where

u_i denotes the predicted value

$\bar{u}$ the actual value

n the number of predictions

5. RESULTS AND DISCUSSIONS

Predicted boride layer thickness

The boride layer thickness's predictions u_{pre} were done initially via the diffusion model u_D then via the equation (18) attained after using the multilinear regression u_L. The results are given in Figure 5 and detailed in Table 5.

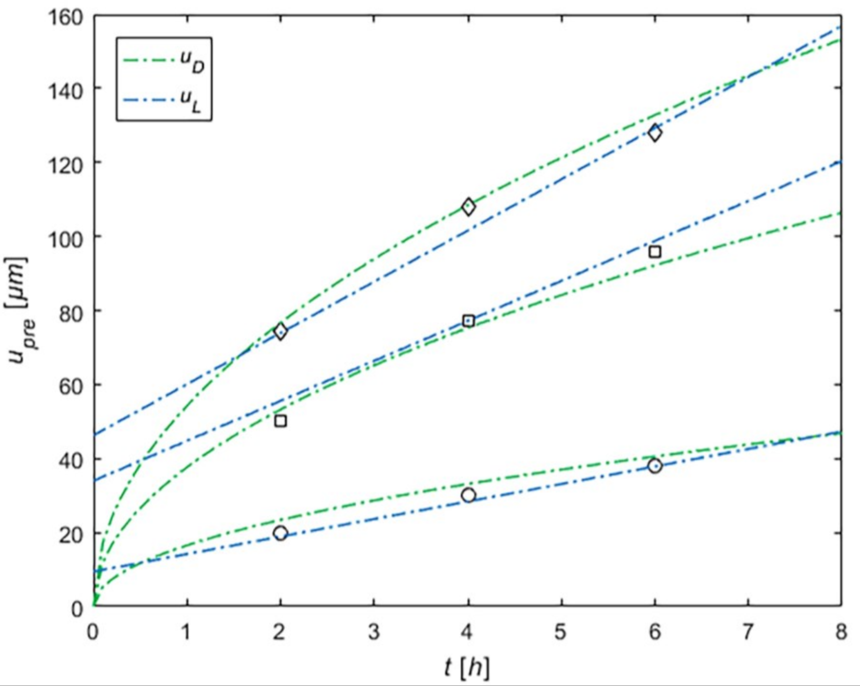

Figure 5. Predicted Fe_2B boride layer thicknesses of each model (Diffusion and Linear)

Slika 5. Predviđene debljine sloja Fe_2B borida za oba modela (difuzioni i linearni)

Figure 5 elucidates that the accuracy of the linear model before 2 h and after 6 h of treatment time is questionable, especially at the beginning of the experiment (0 h), where it is seen that they are unreasonable, having a boride thickness without even beginning the experiment.

Due to such observations, the results of the multilinear regression should not be considered if outside the scope of the studied interval, which was from 2 to 6 hours of treatment time.

Table 5. Predicted Fe_2B boride layer thicknesses values of each model (Diffusion and Linear)

Tabela 5. Predviđene vrednosti debljine sloja borida Fe_2B za oba modela (difuzioni i linearni)

u_{pre} [μm]	T [°C]	t [h] 2	4	6	Residuals		
u_D	850	23.41	33.11	40.55	3.41	3.11	2.55
	950	53.22	75.26	92.17	3.22	-1.74	-3.83
	1000	76.68	108.44	132.82	2.18	0.44	4.82
u_L	850	18.92	28.38	37.85	-1.08	-1.62	-0.15
	950	55.58	77.19	98.80	5.58	0.19	2.80
	1000	73.92	101.60	129.27	-0.58	-6.40	1.27

Nonetheless, as detailed in Table 5, even though the results of the multilinear regression aren't like those of the diffusion model, compared to the experimental data, they were more accurate in seven out of nine predictions. Subsequently, the approach can be conceivable in predicting boride layer thicknesses.

Comparing models

To better understand the accuracy of both approaches, a comparison between both predictions is made with what has been stated in the assessing errors section. The error comparisons were arranged in Table 6.

Ascertaining the results with known error formulas makes it clearer to see the accuracy of both models. Ending up with what's stated in Table 6, an insignificant difference of 0.04 between them in the standard error of the estimate (SEE) and a favorable mean absolute error (MAE) for the linear model over the nonlinear one by 0.62. The other differences concerning the minimal and maximal errors differed insignificantly by 0.29 and 1.58, respectively.

Table 6. Prediction errors of each model (Diffusion and Linear)

Tabela 6. Greške predviđanja za oba modela (difuzioni i linearni)

	u_D	u_L
MAE	2.81	2.19
SEE	3.46	3.50
$Min\ Err$	0.44	0.15
$Max\ Err$	4.82	6.40

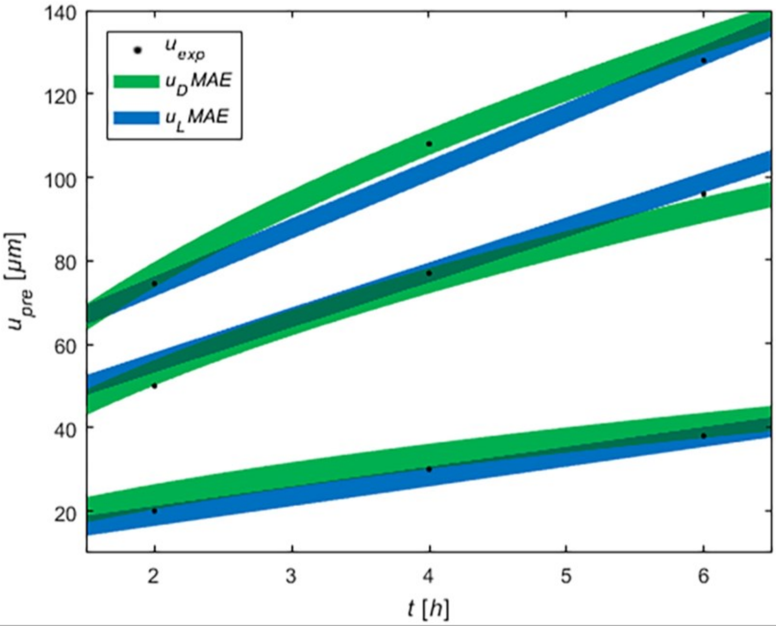

Figure 6. The mean absolute error (MAE) scope in each point for both models

Slika 6. Opseg srednje apsolutne greške (MAE) u svakoj tački za oba modela

To further see the meaning of the mean absolute error (MAE), equation (19), in both models, Figure 6 was established illustrating the scope of error according to it for each model on all nine predicted points, and it is observed that the nonlinear model had five over nine predictions out of the scope but nearing it, while the linear model had only three with two being quite far of the scope indicating that the nonlinear model is slightly more accurate but considering that the linear one uses lesser variables and doesn't consider the incubation time t_0, it can be said that it is still relatively precise in its other predictions.

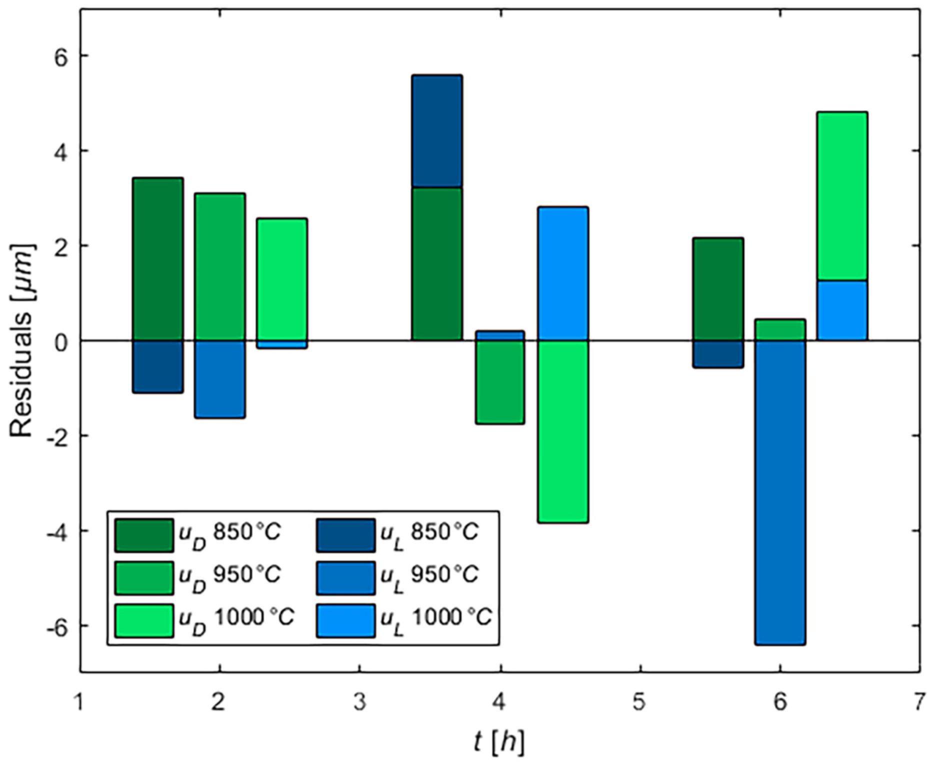

Figure 7. Predictions' residuals on both models

Slika 7. Odstupanja od predviđenih vrednosti za oba modela

Similar to Table 5, Figure 7 efficiently illustrates the resulted residuals from each model's predictions, where it is seen that; the linear model did poorly on two predictions, 4 and 6 h of treatment time at 850 and 950 $°C$ respectively. Nevertheless, it did predict all seven other predictions greater than the nonlinear model (diffusion model).

Additionally, having more negative residuals is better than positive ones because predicting a layer thickness less than its actual value is favored, and that was seen with the linear model contrary to the nonlinear one that had seven out of nine positive residuals, which is detrimental in certain boride layer thicknesses use cases.

With these insignificant differences and such low residuals and errors obtained, it can be stated that both approaches were quite nearing experimental data, making them both comparable and valid models in determining the Fe_2B boride layer thickness.

Iso-thickness diagram

Equation (18), plotted three-dimensionally, displays the obtained predictions of the boride layer thickness u_L as a function of temperature T and time t as a surface plot graphic, Figure 8.

Figure 8 unravels that the influence of temperature is more significant than that of time in the growth of the boride layer thickness. Nevertheless, to better grasp and identify the given predictions from that graphic, they are re-illustrated in Figure 9 as a two-dimensional plot renowned as the iso-thickness diagram.

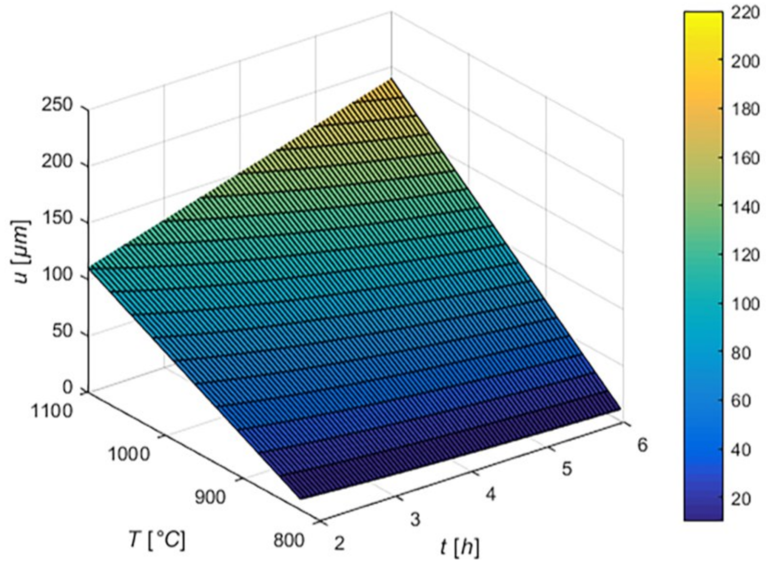

Figure 8. Three-dimensional predictions of the boride layer thickness by temperature and time

Slika 8. Trodimenzionalni prikaz predviđanja debljine boridnog sloja u zavisnosti od temperature i vremena

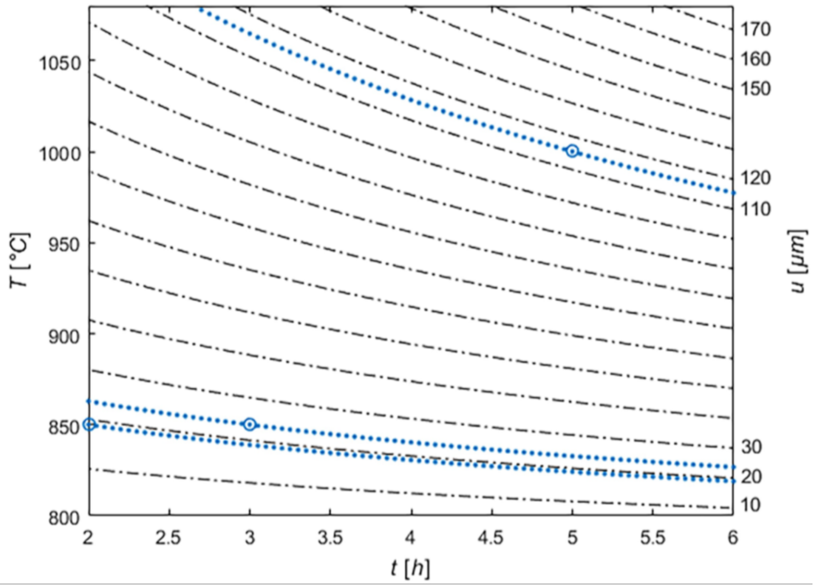

Figure 9. Iso-thickness diagram relating the boride layer thickness to the boriding parameters using the linear model

Slika 9. Dijagram izo-debljine kod linearnog modela koji povezuje debljinu boridnog sloja sa parametrima boronizacije

Table 7. Difference between graphical predictions and calculated ones

Tabela 7. Razlike između grafičkih predviđanja i izračunatih vrednosti

Temperature $[°C]$	850	850	1000
Time $[h]$	2	3	5
u_D $[\mu m]$	23.41	28.68	121.24
Calculated u_L $[\mu m]$	18.92	23.65	115.43
Graphical u_L $[\mu m]$	19	24	115

Other predictions of the boride layer thickness were estimated at 2, 3 and 5 h with a temperature of 850, 850 and 1000 $°C$, respectively. The results are encircled in the iso-thickness diagram within Figure 8 and given in Table 7.

As seen in Table 7, the boride layer thickness can be taken instantly graphically by means of the iso-thickness diagram, with a prediction of near 24 μm for a boriding done with 850 $°C$ for 3 hours, which is found if calculated using the linear regression model's equation (18) as 23.65 μm.

Consequently, the iso-thickness diagram is a valuable tool to instantly determine the temperature T and the treatment time t needed for any desired boride layer thickness u while considering the reading errors.

Alternatively, the same point predicted using the diffusion model, equation (11), gives a boride layer thickness of 28.68 μm, which is higher than what is predicted by the linear model (23.65 μm) but may highly be a higher value than what the actual one might be, and that is from the fact that was discovered in advance (the diffusion model has nearly all predictions over the actual experimental values), so it can therefore be assumed that the actual value may be close to one of them or may be between the predicted values of both models. Given the above, it can be supposed that the two models, linear and nonlinear, are relatively comparable.

6. CONCLUSION

With only two variables from experimental data (temperature and time), a linear-based model has been developed using an advanced multilinear regression, which gave advantageous estimations, making it, from an engineering point of view, a viable approach for Fe_2B boride layer thickness predictions.

Interpolating the predictions of a linear regression beyond the interval of the data sets may lead to detrimental predictions as interpreted, but conversely, exploiting them in the studied interval gave us predictions as accurate as the diffusion model, which was nonlinear and used more variables.

Even though nonlinear models have their advantages in the determination of other factors since they depend on numerous variables, sometimes simplifying things with a linear model can be convenient when the expenses of the research are limited, but keeping in mind that it is more on the practical side than that of a scientifical one.

One of the advantages of the linear model over the nonlinear one is that it often gives predictions that are nearer or inferior to the actual value, and that's favorable when it comes to predicting a layer thickness since expecting less and getting more is beneficial in the end, contrary to predicting a certain thickness and ending up with less which was more the case of the nonlinear model

An iso-thickness diagram is established to instantly identify the temperature and the treatment time required or convenient for any desired Fe_2B boride layer thickness.

While the results are representative of this specific experimental process mentioned and can't be extrapolated on other processes, it was determined that the approach is valid and further studies with it can be carried out on other substrates. Moreover, its potential regarding the kinetics of dual-phased boride layer (FeB and Fe_2B) is to be investigated.

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IZVOD

POREĐENJE LINEARNOG I NELINEARNOG PRISTUPA ZA PREDVIĐANJE DEBLJINA Fe_2B BORIDNIH SLOJEVA

Kinetička ispitivanja boridnih slojeva su usmerena na pokušaje tačnog predviđanja debljine tih slojeva primenom nekih promenljivih uz korišćenje različitih pristupa. U ovom radu su primenjeni pristupi koji se zasnivaju na multilinearnoj regresiji. Pri tome, temperatura T i vreme t su jedine promenljive koje su korišćene u predviđanju debljine sloja borida u, što je značajno sa inženjerske tačke gledišta. Navedeni pristupi koriste eksperimentalne podatke iz procesa boriranja koji je izveden na uzorcima od čelika XC38. Urađeno je poređenje linearnog i nelinearnog modela da bi se odredila valjanost rezultata predviđanja. Nelinearni pristup je poznat kao model difuzije i zasnovan je na drugom Fikovom zakonu. Taj pristup koristi više promenljivih od linearnog pristupa za procenu debljine boriranog sloja. Kao zaključak, izvršeno poređenje je pokazalo da model zasnovanog na linearnoj regresiji može biti pouzdan inženjerski alat za određivanje debljine boridnih slojeva, bez ekstrapolacije rezultata izvan proučavanog opsega.

Ključne riječi: *Multilinearna regresija / Linearna regresija / boronizacija / Boridni sloj / Bor*

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